



## Mathematical prerequisites

The course in **Mathematics for Economics and Business 1** requires knowledge in mathematics based on the curriculum of an Austrian secondary school. As we are an Austrian based university, the curricula are designed according to the Austrian educational standard. This is why we expect applicants to have the knowledge in mathematics which is required from Austrian students in secondary school.

### **This course is based on the contents of the Austrian Matura, which comprises:**

Fundamental Operations with Numbers (Real Numbers, Exponents, Powers, Fractions, etc.), Fundamental Operations with Algebraic Expressions (Terms, Degrees, Grouping, Computations, etc.), Properties of Numbers, Special Products, Factoring (Factorization Procedures, Greatest Common Factor, Least Common Multiple), Fractions (Operations with Fractions, Rational Algebraic Fractions, Complex Fractions), Exponents (Integral Exponents, Roots, Rational Exponents, General Laws of Exponents, etc.), Radicals (Radical Expressions, Laws for Radicals, Operations with Radicals, Rationalizing Binomial Denominators, etc.), Operations with Complex Numbers (Graphical Representation of, Algebraic Operations with Complex Numbers, etc.), Equations (Transformation, Equivalent Equations, Polynomial Equations, etc.), Ratio/Proportion/Variation, Functions and Graphs (Variables, Relations, Functions, Function Notations, Rectangular Coordinate System, Function of Two Variables, Symmetry, Shifts, Scaling, etc.), Linear Equations in One Variable (Linear/Literal Equations), Equations of Lines (Slope of a Line, Parallel and Perpendicular Lines, Slope-Intercept form, Slope-Point Form, Two-point Form, Intercept Form), Simultaneous Linear Equations (Systems of Two/Three Linear Equations), Quadratic Equations in One Variable (Methods of Solving Quadratic Equations, etc.), Systems of Equations Involving Quadratics (Graphical/Algebraic Solution), Inequalities (Principles, Absolute Value Inequalities, Higher Degree Inequalities, Linear Inequalities in Two Variables, Systems of Linear Inequalities, etc.), Polynomial Functions (Polynomial Equations, Zeros of Polynomial Equations, Solving Polynomial Equations, Approximating Real Zeros), Rational Functions (Vertical/Horizontal Asymptotes, Graphing Rational Functions, etc.), Sequences and Series (Arithmetic/Geometric/Harmonic Sequences, Infinite Geometric Series, Means, etc.), Logarithms (Definitions, Laws, Common Logarithms, Natural Logarithms, Use of Tables/Calculators, etc.), Application of Logarithms and Exponents (Simple/Compound Interests, etc.), Permutations and Combinations, The Binomial Theorem, Probability



(Simple/Compound/Binomial/Conditional Probability, Mathematical Expectation),  
Mathematical Induction (Principles of and Proof by Mathematical Induction), Partial  
Fractions (Rational/Proper/Partial Fractions, Identically Equal Polynomials, Fundamental  
Theorem, Finding Decompositions), The Derivative (Slope of a Function, Limits, Derivative  
Function, Adjectives for Functions), Rules for Finding Derivatives  
(Power/Product/Quotient/Chain Rule, Linearity of Derivatives), Curve Sketching (Maxima  
and Minima, First/Second Derivative Test, Concavity and Inflection Points, Asymptotes,  
etc.), Integration (Substitution, Integration by Parts, Rational Functions, etc.).

### **Recommended material for self-study preparation**

1) **Schaum's Outline of College Algebra, 4th Edition (Schaum's Outline Series) by Murray Spiegel, Robert Moyer**

2) **David Guichard, Single and Multivariable Calculus** (available online)

You don't have to go through everything from the book – only the yellow marked parts in the below-mentioned table of content of the book are obligatory prerequisites. The other chapters are helpful but not mandatory.

# Contents

## 1

<b>Analytic Geometry</b>	<b>15</b>
1.1 Lines . . . . .	16
1.2 Distance Between Two Points; Circles . . . . .	21
1.3 Functions . . . . .	22
1.4 Shifts and Dilations . . . . .	27

## 2

<b>Instantaneous Rate of Change: The Derivative</b>	<b>31</b>
2.1 The slope of a function . . . . .	31
2.2 An example . . . . .	36
2.3 Limits . . . . .	38
2.4 The Derivative Function . . . . .	48
2.5 Adjectives For Functions . . . . .	53

### 3

---

#### Rules for Finding Derivatives 57

3.1	The Power Rule	57
3.2	Linearity of the Derivative	60
3.3	The Product Rule	62
3.4	The Quotient Rule	64
3.5	The Chain Rule	67

### 4

---

#### Transcendental Functions 73

4.1	Trigonometric Functions	73
4.2	The Derivative of $\sin x$	76
4.3	A hard limit	77
4.4	The Derivative of $\sin x$ , continued	80
4.5	Derivatives of the Trigonometric Functions	81
4.6	Exponential and Logarithmic functions	82
4.7	Derivatives of the exponential and logarithmic functions	84
4.8	Implicit Differentiation	89
4.9	Inverse Trigonometric Functions	94
4.10	Limits revisited	97
4.11	Hyperbolic Functions	101

### 5

---

#### Curve Sketching 107

5.1	Maxima and Minima	107
5.2	The first derivative test	111
5.3	The second derivative test	112
5.4	Concavity and inflection points	113
5.5	Asymptotes and Other Things to Look For	115

## 6

---

### Applications of the Derivative 119

6.1	Optimization . . . . .	119
6.2	Related Rates . . . . .	131
6.3	Newton's Method . . . . .	139
6.4	Linear Approximations . . . . .	143
6.5	The Mean Value Theorem . . . . .	145

## 7

---

### Integration 149

7.1	Two examples . . . . .	149
7.2	The Fundamental Theorem of Calculus . . . . .	153
7.3	Some Properties of Integrals . . . . .	160

## 8

---

### Techniques of Integration 165

8.1	Substitution . . . . .	166
8.2	Powers of sine and cosine . . . . .	171
8.3	Trigonometric Substitutions . . . . .	173
8.4	Integration by Parts . . . . .	176
8.5	Rational Functions . . . . .	180
8.6	Numerical Integration . . . . .	184
8.7	Additional exercises . . . . .	189

## 9

---

### Applications of Integration 191

9.1	Area between curves . . . . .	191
9.2	Distance, Velocity, Acceleration . . . . .	196
9.3	Volume . . . . .	199
9.4	Average value of a function . . . . .	206
9.5	Work . . . . .	209
9.6	Center of Mass . . . . .	213
9.7	Kinetic energy; improper integrals . . . . .	219
9.8	Probability . . . . .	223
9.9	Arc Length . . . . .	232
9.10	Surface Area . . . . .	234

## 10

---

### Polar Coordinates, Parametric Equations 241

10.1	Polar Coordinates . . . . .	241
10.2	Slopes in polar coordinates . . . . .	245
10.3	Areas in polar coordinates . . . . .	247
10.4	Parametric Equations . . . . .	250
10.5	Calculus with Parametric Equations . . . . .	253

# 11

---

<b>Sequences and Series</b>	<b>257</b>
11.1 Sequences . . . . .	258
11.2 Series . . . . .	264
11.3 The Integral Test . . . . .	268
11.4 Alternating Series . . . . .	273
11.5 Comparison Tests . . . . .	275
11.6 Absolute Convergence . . . . .	278
11.7 The Ratio and Root Tests . . . . .	279
11.8 Power Series . . . . .	282
11.9 Calculus with Power Series . . . . .	285
11.10 Taylor Series . . . . .	286
11.11 Taylor's Theorem . . . . .	290
11.12 Additional exercises . . . . .	296

# 12

---

<b>Three Dimensions</b>	<b>299</b>
12.1 The Coordinate System . . . . .	299
12.2 Vectors . . . . .	302
12.3 The Dot Product . . . . .	307
12.4 The Cross Product . . . . .	313
12.5 Lines and Planes . . . . .	317
12.6 Other Coordinate Systems . . . . .	323

# 13

---

<b>Vector Functions</b>	<b>329</b>
13.1 Space Curves . . . . .	329
13.2 Calculus with vector functions . . . . .	331
13.3 Arc length and curvature . . . . .	339
13.4 Motion along a curve . . . . .	345

## 14

---

### Partial Differentiation 349

14.1	Functions of Several Variables . . . . .	349
14.2	Limits and Continuity . . . . .	353
14.3	Partial Differentiation . . . . .	357
14.4	The Chain Rule . . . . .	363
14.5	Directional Derivatives . . . . .	366
14.6	Higher order derivatives . . . . .	371
14.7	Maxima and minima . . . . .	372
14.8	Lagrange Multipliers . . . . .	377

## 15

---

### Multiple Integration 383

15.1	Volume and Average Height . . . . .	383
15.2	Double Integrals in Cylindrical Coordinates . . . . .	393
15.3	Moment and Center of Mass . . . . .	397
15.4	Surface Area . . . . .	400
15.5	Triple Integrals . . . . .	402
15.6	Cylindrical and Spherical Coordinates . . . . .	405
15.7	Change of Variables . . . . .	409

## 16

---

### Vector Calculus 417

16.1	Vector Fields . . . . .	417
16.2	Line Integrals . . . . .	419
16.3	The Fundamental Theorem of Line Integrals . . . . .	423
16.4	Green's Theorem . . . . .	426
16.5	Divergence and Curl . . . . .	431
16.6	Vector Functions for Surfaces . . . . .	434
16.7	Surface Integrals . . . . .	440
16.8	Stokes's Theorem . . . . .	444
16.9	The Divergence Theorem . . . . .	448



# 17

---

## Differential Equations 453

17.1	First Order Differential Equations . . . . .	454
17.2	First Order Homogeneous Linear Equations . . . . .	458
17.3	First Order Linear Equations . . . . .	461
17.4	Approximation . . . . .	464
17.5	Second Order Homogeneous Equations . . . . .	467
17.6	Second Order Linear Equations . . . . .	471
17.7	Second Order Linear Equations, take two . . . . .	475

# A

---

## Selected Answers 479

# B

---

## Useful Formulas 505

---

## Index 509