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The Sequential Hotelling Game – Closing in on the Equilibrium Outcome

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Abstract

This article presents a theoretical and experimental investigation of a 3-player sequential-entry variant of Hotelling's locational choice model (1929). Martin Osborne and Amoz Kats offer a conjecture about the unique subgame-perfect Nash equilibrium (SPNE) outcome in this game, which I prove for $n = 3$: The first and the last player enter at the median, and the middle player opts out. When used to model political elections, the character of this equilibrium is then related to Duverger's Law, as a two-party system will emerge. Testing this conjecture in the lab reveals that in the beginning, the first and middle players keep out the last player. However, after many repetitions, play converges toward the unique SPNE outcome.

Keywords: Hotelling, Experiment, Sequential, Duverger's Law

1 Introduction

In this paper I analyze a game that was first presented by Martin Osborne and Amoz Kats, which I will henceforth call the sequential Hotelling game:¹

"Each player $1, \dots, n$ chooses a member of the set $[0, 1] \cup \text{OUT}$ (i.e. either chooses a "location" or opts out). The choices are made sequentially (starting with player 1), and every player is perfectly informed at all times. The outcome of the game is determined as follows. After all players have chosen their actions, each player who has chosen a location receives votes from a continuum of citizens; the player who receives the most votes wins. The distribution of citizens' ideal points is nonatomic, with support $[0,1]$. A player who chooses the same position x as $k - 1$ other players obtains the fraction $1/k$ of the votes of all the citizens whose ideal points are closer to x than to any other chosen location. [...] Each player obtains the payoff 0 if she chooses OUT, the payoff $1/k$ if she is among the k players who receive the maximal fraction of votes, and -1 otherwise."^{2,3}

Martin Osborne and Amoz Kats offer a conjecture⁴ about the subgame-perfect Nash-equilibrium (SPNE) outcome in this sequential Hotelling game for an arbitrary number of players n :

Osborne-Kats Conjecture. The sequential Hotelling game has a unique SPNE outcome, in which players 1 and n choose the median location m and all other players choose *OUT*.

This game is interesting in two ways: First, as the only game (to my knowledge) to feature a first-mover and a last-mover advantage simultaneously, it is unique in a game theoretical sense. Second, the specific character of the equilibrium is related to Duverger's Law:⁵ When the game is interpreted as modeling the location decisions of political actors

¹ The sequential Hotelling game is a variant of Hotelling's locational choice model (1929) and its refinement by Duverger (1954).

² In the original game description the term "position" is used instead of the term "location"; this was changed for consistency reasons because I use the term location in the experiment.

³ Freely available on Martin Osborne's homepage under <http://www.economics.utoronto.ca/osborne/research/CONJECT.HTM>

⁴ A proof for the general case does not exist yet.

⁵ A plurality voting system often leads to a two-party system; Duverger (1954).

on a left-right spectrum, a two-party system will emerge.

I analyze the three-player variant of this game, for which I derive all SPNE and prove the conjecture for $n = 3$.⁶ Furthermore, I implement the game under laboratory conditions and conduct an experiment, in which the Osborne-Kats conjecture makes a strong behavioral prediction in the lab, as the SPNE outcome is unique.⁷ Subjects play the sequential Hotelling game as a finitely repeated game, with the treatments differing (mostly) in game length.

Initial play in the experiment leads to an outcome that favors players 1 and 2, which is not in accordance with the SPNE in the sequential Hotelling game. As the game progresses, however, players learn to best respond, resulting in convergence toward the unique SPNE outcome after many repetitions of the game.

As Hotelling's original locational choice model (1929) and its derivations are highly relevant in the fields of political science and industrial organization, numerous theory papers exist on this topic; see Osborne (1995) for a general review. While the simultaneous move case and its variants have also been analyzed experimentally,⁸ changes in timing, i.e. the sequential entry case, have only been analyzed theoretically,⁹ so never empirically or experimentally. Therefore, my contribution to the literature is a theoretical and experimental investigation of the Hotelling game with sequential entry.

⁶ Detailed arguments for the unique SPNE outcome in the special case of $n = 3$ were already made in Osborne (2004) and on <http://www.economics.utoronto.ca/osborne/research/ARG.HTM>, but to the author's knowledge a formal proof does not exist in the literature.

⁷ The lab implementation is not exactly the same as the sequential Hotelling game, as the voter support is continuous in theory but must necessarily be discrete in the lab; more on that in Section 2.2.

⁸ See Brown-Kruse, Cronshaw and Schenk (1993), Brown-Kruse and Schenk (2000), Collins and Sherstyuk (2000), Huck, Müller and Vriend (2000), Barreda-Tarrazona et al. (2011) and Kephart and Friedman (2015).

⁹ See Prescott and Visscher (1977), Neven (1987), Eiselt and Laporte (1997), Osborne (2004), Rabas (2011), Bandyopadhyay et al. (2016), as well as Kress and Pesch (2012) for an overview.

2 Theory

2.1 The Sequential Hotelling Game

Each of the players $i = 1, \dots, n$ chooses as his action a_i an element of the set $[0, 1] \cup \{OUT\}$. That is, each player either chooses a location ($a_i \in [0, 1] \setminus OUT$) or opts out ($a_i = OUT$). The choices are made sequentially starting with player 1 and ending with player n , and every player observes all previous choices.

The outcome of the game is then determined as follows: After all players have chosen their actions, each player i who has chosen a location $a_i \neq OUT$ receives vote shares $v_i(a_1, a_2, a_3) \in (0, 1]$, where $\sum_{i=1}^n v_i = 1$, from a continuum of voters. The player(s) who receive(s) the highest vote shares $v^{max} = \max(v_1, v_2, \dots, v_n)$ win(s); a player who has chosen $a_i = OUT$ receives no votes and cannot win. Each voter simply votes for the player whose chosen location $a_i \neq OUT$ is closest to the voter's ideal location, and the distribution of voters' ideal locations is uniform along the interval $[0, 1]$ and nonatomic.¹⁰ Furthermore, if a player i has chosen the same location a_i as $z - 1$ other players, he obtains the fraction $v_i = 1/z$ of the votes of all voters whose ideal location is closer to a_i than to any other chosen location.

Each player i then obtains payoff π_i according to the following formula, where s denotes the number of players with $v_i = v^{max}$, i.e. $s = |\{i \in \{1, \dots, n\} \mid v_i = v^{max}\}|$:

$$\pi_i = \begin{cases} 0 & \text{if } a_i = OUT \\ \frac{1}{s} & \text{if } a_i \in [0, 1] \text{ and } v_i = v_{\max} \\ -1 & \text{if } a_i \in [0, 1] \text{ and } v_i < v_{\max} \end{cases}$$

That is, each player obtains payoff 0 if he chooses $a_i = OUT$, payoff $1/s$ if he is among the s players who receive the maximal share of votes, and -1 if there exists a player who has more votes. This implies that each player wants to enter the competition if and only if he has some chance of winning.

¹⁰ This means that if a voter's favorite location is x^* , he is indifferent between the locations $x^* - s$ and $x^* + s$. This also means that the voters do not vote strategically, and voting is sincere.

For readability, I also introduce two definitions:

Definition 1. I define “choosing a location” and “entering the game” as choosing an $a_i \neq OUT$. Furthermore, in the sequential Hotelling game, I define choosing an $a_i < t$ as $a_i \in [0, t)$. Finally, if $a_i = OUT$, player i “stays out of the game”.

Definition 2. In the Sequential Hotelling Game, I call a player “winning” if he has payoff $\pi_i > 0$, and I call a player “losing” if he has payoff $\pi_i = -1$. Furthermore, a player “wins alone” if he has strictly more votes than any other player.

2.1.1 The SPNE for the Sequential Hotelling Game

In this paper, I will look at the case of $n = 3$. The different subgame-perfect Nash-equilibria for $n = 3$ are characterized as follows:¹¹

$$a_1^* = 0.5, a_2^*(a_1) = \begin{cases} 0.5 & \text{if } a_1 = OUT \\ \left[\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1 \right] & \text{if } a_1 < \frac{1}{6} \\ \left[\frac{2-a_1}{3}, 1 - a_1 \right] & \text{if } \frac{1}{6} \leq a_1 \leq 0.5 \\ OUT & \text{if } a_1 = 0.5 \end{cases} \quad (1)$$

while player 3 chooses according to the following rules:

1. If the set $A = \{a_3 | v_3 > \max(v_1, v_2)\}$ is nonempty, i.e. if player 3 can attain $v_3 > \max(v_1, v_2)$ by choosing some $a_3 \in [0, 1]$ he chooses one of these payoff-maximizing choices.
2. If set A is empty and the set $B = \{a_3 | v_3 = \max(v_1, v_2)\}$ is nonempty, i.e. player 3 can attain $v_3 = \max(v_1, v_2)$ by choosing some $a_3 \in [0, 1]$, he chooses one of them.
3. If both sets A and B are empty, $a_3 = OUT$.

¹¹ Because of the symmetrical nature of the game around the median location 0.5, there are certain symmetries in this game. In general, if we make any statement concerning outcome, vote shares or best responses about a choice triple (a_1, a_2, a_3) , the same statement is still true if we consider the choice triple $(1 - a_1, 1 - a_2, 1 - a_3)$ (here I define for $a_i = OUT$ that $1 - a_i = OUT$). Or in other words, $\pi_i(a_1, a_2, a_3) = \pi_i(1 - a_1, 1 - a_2, 1 - a_3)$ and $v_i(a_1, a_2, a_3) = v_i(1 - a_1, 1 - a_2, 1 - a_3) \forall i$. For best responses, it holds that if a^* is a best response to a_1 (given a^*), then $1 - a^*$ is a best response to $1 - a_1$ (given $1 - a^*$). For player 3, if a^* is a best response to (a_1, a_2) , $1 - a^*$ is a best response to $(1 - a_1, 1 - a_2)$. Therefore, cases of $a_1 = 0.5 + s$ are symmetrical to $a_1 = 0.5 - s$ (for $s \leq 0.5$), and I can omit all cases $a_1 > 0.5$ w.l.o.g.

It is important to note that while the subgame-perfect Nash-equilibrium outcome is unique, the equilibria themselves are not.¹² In equation (1), we can clearly see why this is the case: After many histories, the best responses by players 2 and 3 are not unique off the equilibrium path. We see for example that for player 2, there is an infinite number of best responses given $a_1 < \frac{1}{6}$. The same is true for player 3, as there are many cases where there is a range of best responses following (a_1, a_2) , giving an infinite number of subgame-perfect Nash equilibria.

The important fact is that according to any SPNE, play along the equilibrium path consists of $a_1 = 0.5$, $a_2 = OUT$, $a_3 = 0.5$, in accordance with the Osborne-Kats-conjecture. The derivation of the above SPNE as well as the uniqueness of the outcome can be found in appendix A.1.

2.2 Lab Implementation

The problem with the transition of the sequential Hotelling game as described above to a laboratory environment is that a player's action space is continuous in theory, but necessarily discrete in the lab. Furthermore, it is crucial that player 3 has the option to choose an a_3 closer to the median than player 2.¹³

I solve this problem by giving subjects different discrete action spaces depending on their positions, i.e. the order in which the players choose their actions $a_i \in X_i$:

- A player in position 1 can choose from $X_1 = \{OUT, 1, 9, 17, 25, 33, 41, 49\}$
- A player in position 2 can choose from $X_2 = \{OUT, 1, 5, 9, \dots, 41, 45, 49\}$
- A player in position 3 can choose from $X_3 = \{OUT, 1, 3, 5, \dots, 45, 47, 49\}$

In this way, subjects who choose later have more options than subjects who choose

¹² This fact is also pointed out in Osborne (2004).

¹³ If this were not the case, the unique SPNE outcome for $n = 3$ would be given by $a_1^* = OUT$, $a_2^* = 0.5$, $a_3^* = 0.5$, a different equilibrium outcome. The reasoning for this is the following: Assuming player 1 chooses $a_1 = 0.5$, player 2 can adopt the location $a_2 = 0.5 - c$ closest to 0.5 that is possible, while player 3 now has no option to locate closer to the middle than player 2, so he chooses $a_3 = OUT$. Therefore, player 1 would not enter the game, and the other 2 players locate at the median.

earlier. The set $X = \{1, 2, 3, \dots, 47, 48, 49\}$ contains all locations in the game. The locations available to be chosen by each player i are defined as $X_i \setminus \{OUT\}$.

The outcome of the game is determined similarly as in the sequential Hotelling game: After all players have chosen their actions, each player who has chosen a location, i.e. $a_i \in X_i \setminus \{OUT\}$, receives a number of points $v_i(a_1, a_2, a_3) \in (0, 48]$, while a player who chooses $a_i = OUT$ receives no points, i.e. $v_i = 0$. Each location $x \in [2, 3, 4, \dots, 46, 47, 48]$ is worth one point, and the locations on the edges ($x = 1$ and $x = 49$) are worth half a point. Therefore, the sum of points to be gained is 48. Each player receives points from each location x that is closer to his chosen location a_i than to any other location that was chosen by another player. If a player i has chosen the same location a_i as $z - 1$ other players, he obtains the fraction $v_i = 1/z$ of the points from locations that are closer to a_i than to any other chosen location. Furthermore, if an unchosen location is equally distant between two chosen locations a_i and a_j , the point for this unchosen location is split evenly between the players who have chosen a_i and a_j . The player(s) who receive(s) the largest share of points $v^{max} = \max(v_1, v_2, v_3)$ win(s), given that $v^{max} \neq 0$.

Each player i then obtains payoff π_i according to the following formula, where s denotes the number of players with $v_i = v^{max}$ who choose a location, i.e. $s = |\{i \in \{1, 2, 3\} \mid v_i = v^{max} \cap v_i \neq 0\}|$:

$$\pi_i = \begin{cases} 0.25 & \text{if } v_i = 0 \\ \frac{2}{s} & \text{if } v_i = v^{max} \text{ and } v_i \neq 0 \\ 0.05 & \text{if } v_i < v^{max} \text{ and } v_i \neq 0 \end{cases}$$

That is, each player gets 0.25 if he chooses $a_i = OUT$, payoff $2/s$ if the player is among the s players who receive the maximal share of points, and 0.05 if there exists a player who has more points.¹⁴

The definitions of winning, losing and entering the game for the lab game are as follows:

Definition 3. I define choosing an $a_i < t$ as choosing an $a_i \in \{X_i \cap [1, t)\}$ in the lab game.¹⁵

¹⁴ For the lab game to have qualitatively the same incentives as the sequential Hotelling game, two conditions must be fulfilled as far as the payoff parameters are concerned: If player i wins, his payoff must always be higher than if he chooses $a_i = OUT$ (which is satisfied here, as $2/n$ is always higher than 0.25), and if a player chooses OUT he must have a higher payoff than if he loses, which is also satisfied.

¹⁵ For example, if player 1 chooses an $a_1 < 17$, he either chooses $a_1 = 1$ or $a_1 = 9$.

Definition 4. In the lab game, I call a player “winning” if he has payoff $\pi_i > 0.25$, and I call a player “losing” if he has payoff $\pi_i = 0.05$. Furthermore, a player “wins alone” if he has strictly more votes than any other player.

Definition 5. In the lab game, I say that a player “enters the game” if that player chooses any location, i.e. $a_i \in X_i$ *OUT*, and a player “stays out of the game” if $a_i = OUT$.

A The SPNE in the Lab Implementation

Analogous to the sequential Hotelling game, the SPNE for the lab implementation is not unique, but the SPNE outcome is. The SPNE for the lab implementation is characterized by¹⁶

$$a_1^* = 25, a_2^*(a_1) = \begin{cases} 25 & \text{if } a_1 = OUT \\ 37 & \text{if } a_1 = 1 \\ \{33, 37\} & \text{if } a_1 = 9 \\ 29 & \text{if } a_1 = 17 \\ OUT & \text{if } a_1 = 25, \end{cases}$$

While player 3 chooses according to the following rule:

1. If the set $A = \{a_3 | v_3 > \max(v_1, v_2)\}$ is nonempty, i.e. if player 3 can attain $v_3 > \max(v_1, v_2)$ by choosing some $a_3 \in X_3$, he chooses one of these payoff-maximizing choices.
2. If set A is empty and the set $B = \{a_3 | v_3 = \max(v_1, v_2)\}$ is nonempty, i.e. player 3 can attain $v_3 = \max(v_1, v_2)$ by choosing some $a_3 \in X_3$, he chooses one of them.
3. If both sets A and B are empty, $a_3 = OUT$.

¹⁶ Similar to footnote 11, because of the symmetrical nature of the game around the median location 25, there are certain symmetries in this game. In general, if we make any statement concerning outcome, vote shares or best responses about a choice triple (a_1, a_2, a_3) , the same statement is still true if we consider the choice triple $(50 - a_1, 50 - a_2, 50 - a_3)$ (here I define for $a_i = OUT$ that $50 - a_i = OUT$). Or in other words, $\pi_i(a_1, a_2, a_3) = \pi_i(50 - a_1, 50 - a_2, 50 - a_3)$ and $v_i(a_1, a_2, a_3) = v_i(50 - a_1, 50 - a_2, 50 - a_3) \forall i$. For best responses, if a^* is a best response to a_1 (given a^*), then $50 - a^*$ is a best response to $50 - a_1$ (given $50 - a^*$). For player 3, if a^* is a best response to (a_1, a_2) , $50 - a^*$ is a best response to $(50 - a_1, 50 - a_2)$. Therefore, cases of $a_1 = 25 + s$ are symmetrical to $a_1 = 25 - s$ (for $s < 25$), and I can omit all cases $a_1 > 25$ w.l.o.g.

Akin to the sequential Hotelling game, we easily see that the SPNE is not unique, as player 2 can choose to play either $a_2 = 33$ or $a_2 = 37$ following $a_1 = 25$, as both choices are best responses. Furthermore, after many histories, the best response for player 3 is not unique off the equilibrium path. The complete SPNE, including all best responses given all histories for all players, can be found in appendix A.2.

With these action spaces and parameter choices, I chose an implementation for the lab that changes as little as possible compared to the sequential Hotelling game, while preserving the equilibrium prediction and also the intuition behind it.

The intuition for the SPNE in terms of the lab game (and therefore similarly in the sequential Hotelling game) is the following:

Consider the case of $a_1 < 25$, i.e. player 1 choosing to enter the game but not at the median location. In this case, player 2 best responds by locating to the right of the median in such a way that it is not possible for player 3 to choose a location such that $v_3 \geq \max(v_1, v_2)$, and such that $v_2 > v_1$ if $a_3 = OUT$. This means that player 2 can guarantee himself a win in all subgames following $a_1 < 25$. Therefore, as player 1 can guarantee himself the higher payoff of 0.25 by choosing OUT , $a_1 < 25$ cannot be part of an SPNE.

Next, consider the case of $a_1 = 25$. Then the best response for player 2 is to play $a_2 = OUT$: If player 2 chooses any location to the left of the median or the median itself, player 3 chooses a location close to and to the right of the median and wins. Therefore, player 2 will play $a_2 = OUT$. Now player 3 can only tie with player 1 for first place by choosing $a_3 = 25$ (i.e. the median), which is profit maximizing for him. Player 1 is therefore better off choosing $a_1 = 25$ than $a_1 = OUT$ and splits the win with player 3.

So when we put this together, play along the equilibrium path consists of $a_1 = 25$, $a_2 = OUT$ and $a_3 = 25$, which is in accordance with the Osborne-Katz-conjecture, as the first and the last player enter at the median and the middle player stays out.

2.3 The Experiment

I conducted eleven sessions at the Vienna Center for Experimental Economics (VCEE) with 132 subjects. Sessions lasted about 2 hours on average. The range of earnings was between €6 and €53, with an average payment of about €26. The experiment was programmed and conducted with the software z-Tree (Fischbacher (2007)), and ORSEE (Greiner (2004)) was used for recruiting subjects.

2.3.1 Parameters

The game is played over 24, 48 or 72 rounds, depending on the treatment. In each round, subjects were randomly rematched with two other subjects from the matching pool of 12 subjects to form groups of three. Each subject was then randomly assigned a position (i.e. the order in which they would act) within these groups of three subjects, with the constraint that after all rounds, every subject had been in all positions the same number of times. Subjects were assigned new positions after each round instead of keeping their positions fixed for two reasons: First, it would be unfair to subjects in middle positions in terms of payoffs, as there is a first-mover and last-mover advantage in this game; second, I thought that subjects would learn faster if they experienced the game from all player positions.

Subjects determined the locations they wanted to choose by means of a slider that only lets them choose locations that are in their action space, and there is a button labeled "No Location" for choosing $a_i = \text{OUT}$. Subjects were able to see all actions of their two group members' previous choices in this round, i.e. player 2 sees player 1's choice when he makes his decision and player 3 sees both choices from players 1 and 2. This was clearly represented on the slider and in written form on the decision screen.

After each round, subjects saw a detailed feedback screen, indicating all chosen locations by all three group members and their respective points in this round, as well as feedback on all players' payoffs in their group in this round. A subject's total payoff in the experiment was the sum of all payoffs from all rounds in Euro, i.e. the exchange rate of points in the game to Euro was 1:1. Note that with these parameter choices losses are not possible in the experiment, as the minimum payoff a subject can get in each round is €0.05.

A Instructions and Questionnaire

The experiment started with on-screen instructions where neutral framing was used. Instructions were followed by control questions; see Section A.3 in the appendix. After the 24, 48 or 72 game rounds, a short questionnaire concluded the experiment.¹⁷ In addition, there were questions of the form “When you were in position 1, what did you do and why?” for all positions, as well as more subtle questions; see Section A in the appendix.

B Treatments

The baseline treatment 24R corresponds to the lab game described in Section 2.2 played over 24 rounds, the second treatment changes the tiebreaking rule and the third and fourth treatment increase the number of rounds played; this is represented in Table 1. See the next section for details.

Table 1: *Treatments*

Treatment	Rounds	Observations	Subjects	Tiebreak rule
24R	24	288	36	standard
24R+A	24	288	36	alternative
48R	48	576	36	standard
72R	72	864	24	standard

Notes: The standard tiebreak rule corresponds to the one in Section 2.2, namely that if two or more players have the same maximal number of votes, they split the prize evenly. The alternative rule is that these players enter a lottery, where one of them gets the whole prize.

¹⁷ I elicited standard socioeconomics like age, gender, income and highest education, as well as a standard Cognitive Reflection Test (Frederick, 2005). On top of that I also used a short incentivized measure for social preferences, namely a variant of a test proposed in Thibaut and Kelley (1959).

3 Results

For simplicity I will denote choice triples by (a_1, a_2, a_3) ,¹⁸ and conditional choices will be written as $a_2(a_1 = a) = c$ for player 2 and $a_3(a_1 = a, a_2 = b) = c$ for player 3.¹⁹

As we have already seen in footnote 16 in Section A, due to the symmetric nature of the game around the median location of 25, many choice triples are symmetric and lead to the same payoff and vote shares, and are therefore handled as the same observation. So to merge all symmetric observations together, I transform all cases of $(a_1 > 25, a_2, a_3)$ into $(50 - a_1, 50 - a_2, 50 - a_3)$; I transform all cases of $(a_1, a_2 > 25, a_3)$ into $(50 - a_1, 50 - a_2, 50 - a_3)$ if $a_1 \in \{OUT, 25\}$; and I transform all cases of $(a_1, a_2, a_3 > 25)$ into $(50 - a_1, 50 - a_2, 50 - a_3)$ if $a_1 \in \{OUT, 25\}$ and $a_2 \in \{OUT, 25\}$. Basically, any observation where a player is the first to choose a location a_i that is not the median is equivalent to that player choosing location $50 - a_i$, and choices made by following players are adjusted accordingly.²⁰

3.1 Treatment 24R - Baseline

In this treatment, the game described in Section 2.2 was played for 24 rounds. Three sessions with 12 subjects each were conducted, resulting in 288 observations (24 rounds x 4 groups x 3 sessions).

Figure 1 shows the distribution of locational choice triples, split into rounds 1-12 and 13-24. Note first that there is an absence of the unique SPNE outcome (25, OUT, 25) in rounds 1-12, and it is observed only two times in rounds 13-24 across all three sessions. In the first 12 rounds, the observation (17, 33, OUT) is most common. In this case, players 1 and 2 “corner the market” and win, which means they locate in such a way that player 3 has no possibility to enter and win the game. We can also see in Figure 1 that in rounds 13-24, (17, 33, OUT) is still the most frequent observation, but we also see a significant rise

¹⁸ For example, (17, 33, OUT) means that the player in position 1 chose location 17, the player in position 2 chose location 33 after observing player 1’s choice of location 17, and the player in position 3 did not choose a location.

¹⁹ For example, if player 2 chooses location 33 after observing player 1’s choice of location 17, I would write $a_2(a_1 = 17) = 33$. If player 3 then observes both these choices and chooses OUT, I would write $a_3(a_1 = 17, a_2 = 33) = OUT$.

²⁰ For example, $(25, 25, 23) \cong (25, 25, 27)$, $(OUT, 9, 33) \cong (OUT, 41, 17)$ and $(25, OUT, 1) \cong (25, OUT, 49)$, where \cong means that they are handled as the same observation.

of (17, 29, *OUT*).²¹ The response by player 2 of $a_2 = 29$ given $a_1 = 17$ is in line with the SPNE, as player 3 can still not find a location to win or share the win, but player 2 gets more points than player 1 and wins alone.

Overall, players in position 3 chose according to the SPNE a majority of the time (67% across all 24 rounds); this high frequency is not surprising, as player 3 has no uncertainty about the behavior of the other players. Players in position 2 chose according to the SPNE only 18% of the time. Player 1 chose according to the SPNE (i.e. $a_1 = 25$) in 28% of observations. The choice of player 1 to favor $a_1 = 17$, however, was payoff-maximizing in most cases given the behavior of players 2 and 3: The payoff of players in position 1 who choose $a_1 = 17$ is about three times higher than the payoff of those who play $a_1 = 25$; more on this in Section 3.3.

3.2 Treatment 24R+A - different tiebreaking

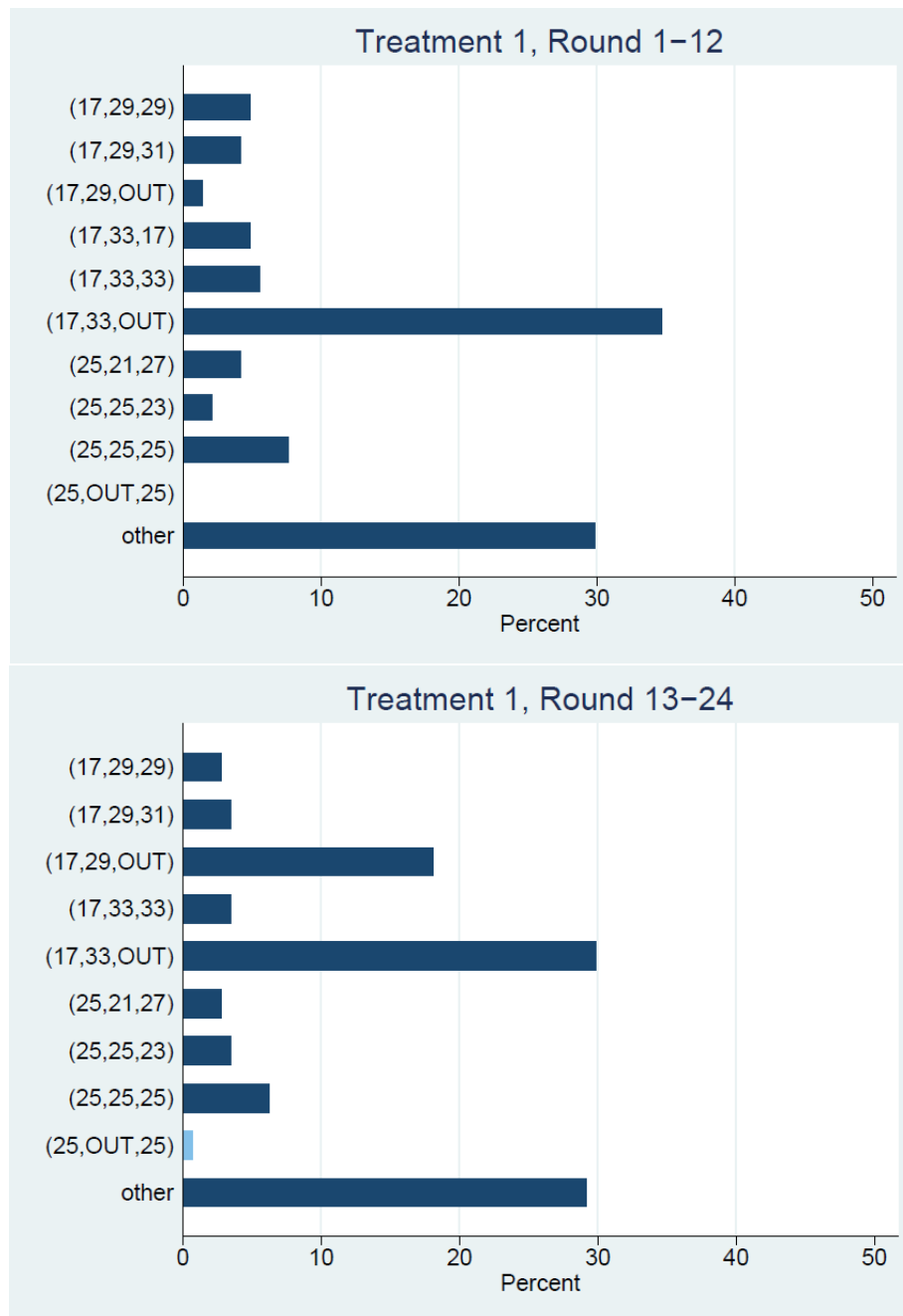
We have seen in treatment 24R that there is next to no play of the unique SPNE outcome, while (17, 33, *OUT*) is the dominating outcome. Reciprocity might be the cause of this behavior, as it could drive player 2 to play $a_2 = 33$ given $a_1 = 17$ to share the win with player 1: If player 2 realizes that he cannot win if $a_1 = 25$ (see Section A), he might be thankful to player 1 and share the win with him while still keeping player 3 out of the game.²²

In treatment 24R+A, the next step was to include a different tiebreaking rule than in treatment 24R to deter reciprocal behavior and give the SPNE a better shot. Consequently, the tiebreaking rule in the baseline (i.e. if two or more players have the same number of points in a round, they split the prize evenly) was changed to a rule that undermines reciprocal and egalitarian incentives: If two or more players have the same number of points at the end of a round, one of them gets the full prize

²¹ The difference in (17, 29, *OUT*) in the first and second half of the game is significant at $p = 0.003$; OLS regression on the individual level with standard errors clustered by session, where the dependent variable is the fraction of plays of $a_2 = 29$ given $a_1 = 17$ by an individual subject, and the independent variable is a dummy for round 13-24; for the detailed specification, see footnote 26.

²² An incentivized measure for social preferences was elicited in the experiment. The test of whether player 2's response of $a_2 = 33$ given $a_1 = 17$ occurs more frequently for subjects with high social preferences was, however, inconclusive.

Figure 1 *Learning in Treatment 24R*



Notes: Locational choice triples on the y-axis, frequency in percent on the x-axis; only 2 observations of the unique SPNE outcome; other triples with less than 20 observations are merged in “other”; N= 288.

and the other(s) get(s) nothing, with equal chances.²³ With this new tiebreaking rule, players in position 2 who choose $a_2 = 33$ given $a_1 = 17$ will now not split the prize with player 1 but rather enter a lottery for the prize. According to the standard tiebreaking rule, player 2 could (almost) be sure that he would split the prize with player 1; now, one of the two will get the whole prize, and players motivated by reciprocity will be deterred from choosing $a_2 = 33$ given $a_1 = 17$ due to the fact that if risk is involved compared to sure decisions, generous giving is significantly reduced. Brock et al. (2013) observe this behavior in dictator games, and player 2 faces a choice akin to the dictator in these games, as he can either (in a majority of cases) take the whole prize for himself, or split it with player 1.²⁴ On the other hand, if reciprocity does not explain this behavior, the new tiebreaking rule would not change behavior. As in the baseline, three sessions were conducted with new subjects, giving 288 observations.

Figure 2 shows that there are more observations of the SPNE outcome (highlighted) than in treatment 24R, although not significantly so (see Table 6 in the appendix). Overall choices did not change substantially compared to the baseline: The dominating presence of $(17, 33, OUT)$ and the rise in $(17, 29, OUT)$ can still be observed.²⁵ If anything, reciprocal behavior got stronger, as we observe fewer plays of $(17, 29, OUT)$ than in the baseline and more plays of $(25, 25, 25)$, which corresponds to a three-way tie. In fact, I detect no significant differences between treatment 24R and 24R+A in any variables. The corresponding tests based on regressions can be found in appendix A.4, Table 6.

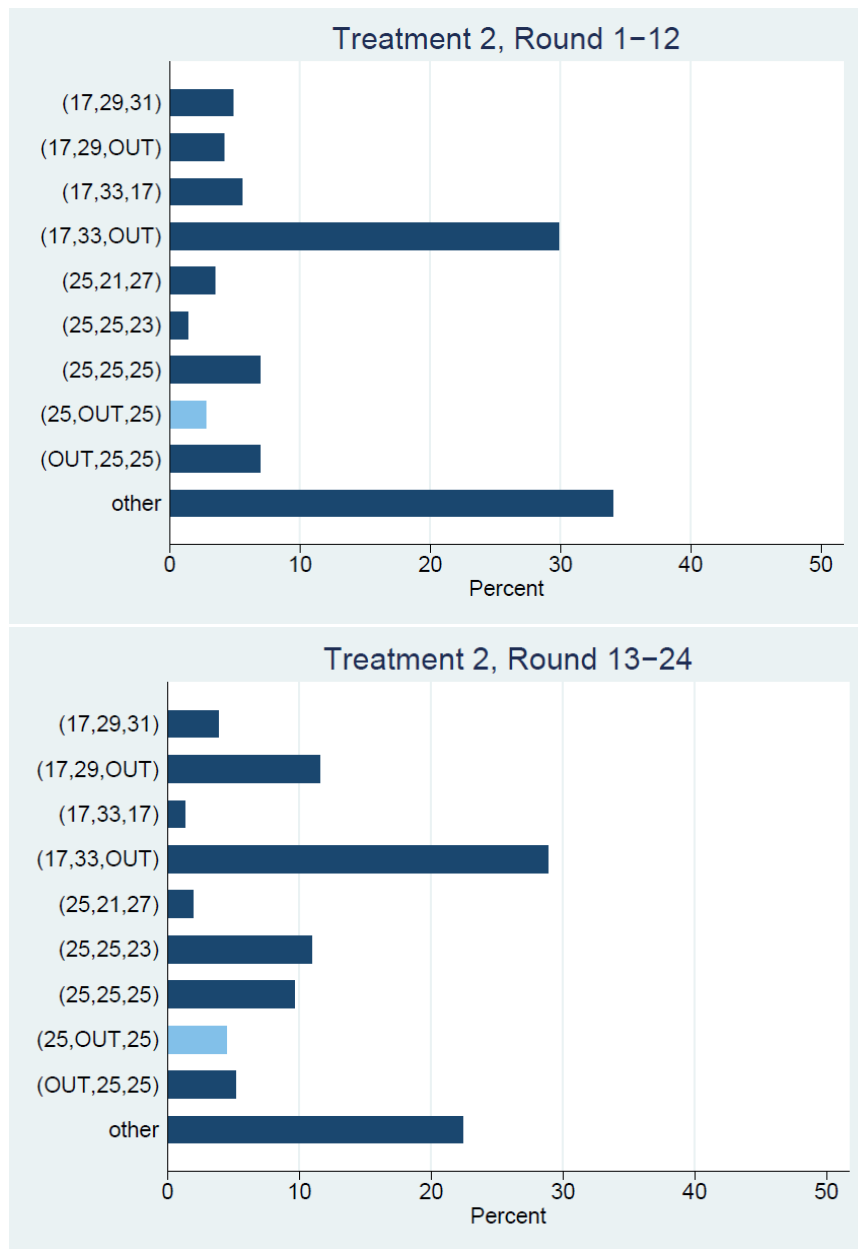
The number of plays according to the SPNE is also similar to the baseline: 28% of players in position 1, 21% of players in position 2 and 65% of players in position 3 chose actions in accordance with the SPNE. I conclude that the results from treatment 24R+A do not significantly deviate from those of treatment 24R. This, and the fact that there are no references in the questionnaires that players in position 2 play $a_2(a_1 = 17) = 33$ due to

²³ This new tiebreaking rule does not change the SPNE assuming risk neutrality, as in expectation the payoffs remain the same.

²⁴ Note that also in the case of $(25, 25, 25)$, which makes up about 7% of the baseline sample, reciprocal behavior might be the cause of player 3's choice to split the prize with players 1 and 2 instead of winning alone by choosing any location close to 25.

²⁵ As in treatment 24R, an OLS regression on the individual level clustered by session shows that the number of plays of $a_2(a_1 = 17) = 29$ rises significantly in the second half of the game, $p = 0.044$; see footnote 26 for a detailed specification.

Figure 2 *Learning in Treatment 24R+A*



Notes: Locational choice triples on the y-axis, frequency in percent on the x-axis; triples with less than 20 observations are merged in “other”; unique SPNE outcome highlighted; N=288.

reciprocity, is a strong indication that prosocial preferences are unlikely to explain behavior in the treatment 24R.

3.3 Treatments 48R and 72R - Giving Subjects more time to learn

In treatment 48R the number of rounds of the game was increased to 48 and in treatment 72R to 72 rounds. As we saw no significantly different behavior with the tiebreaking rule in treatment 24R+A, the tiebreaking rule of the baseline (if two or more subjects have the same number of points at the end of a round, all of them split the prize evenly) was reinstated for treatments 48R and 72R.

The design choice to increase the length of the game follows from the observations made in the first two treatments: Learning could be observed, albeit slowly and not directly towards the SPNE. The extent of learning that can be observed in treatments 24R and 24R+A is summarized in Table 2, where I pool all data from these two treatments and compare behavior in rounds 13 – 24 to behavior in rounds 1 – 12. Play according to the SPNE rose from rounds 1 – 12 to rounds 13 – 24 for player 3 (56% vs. 66%, $p = .049$, OLS regression²⁶), and players in position 2 slowly learn to best respond to $a_1 = 17$ with $a_2(a_1 = 17) = 29$ (12% vs. 23%, $p = .021$, OLS regression, for specification see footnote 26). No significant learning can be observed for player 1, but this is not surprising as the subjects face a backward induction problem, so it makes sense that learning starts with players 2 and

²⁶ The regression that is used for testing has the form $Y_{ij} = \beta_0 + \beta_1 * Dummy + c_{ij}$, where Y is the dependent variable of interest and *Dummy* represents a dummy variable to be tested that takes values $j = \{0, 1\}$, while i is the subject index. The dependent variable is a mean that is calculated individually for each subject i , and standard errors are clustered by session. The null hypothesis is $\beta_1 = 0$.

For example, if I want to test whether the frequency of SPNE play by player 1 (i.e. $a_1 = 25$) increased in rounds 13 – 24 compared to rounds 1 – 12, I calculate $Y_{i,0}$ and $Y_{i,1}$ for all i , where $Y_{i,j} = (\text{number of times subject } i \text{ played } a_1 = 25) / (\text{number of times subject } i \text{ was in position 1})$, and where $j = 1$ (0) if the round number is 13 – 24 (1 – 12). I then run the above regression with the dummy variable taking value 1 if the round number is 13 – 24, and 0 otherwise.

If I want to test treatment differences, I construct a treatment dummy. For dependent variables which are conditional, e.g. $a_2(a_1 = 17) = 29$, $Y_{i,j} = (\text{number of times subject } i \text{ played } a_2(a_1 = 17) = 29) / (\text{number of times subject } i \text{ was in position 2 and } a_1 = 17)$. To check SPNE play in general for players 2 (or 3), the dependent variable is $Y_{i,j} = (\text{number of times subject } i \text{ played a best response}) / (\text{number of times subject } i \text{ was in position 2 (or 3)})$; best responses can be found in Table 5 in the appendix.

3.

Table 2: *Learning in Treatments 24R and 24R+A*

	Rounds 1-12	Rounds 13-24	p-value
SPNE play by Player 1	21%	28%	.259
SPNE play by Player 2	17%	21%	.442
SPNE play by Player 3	56%	66%	.049
$a_2(a_1 = 17) = 29$	12%	23%	.021

Notes: P-value from an OLS regression, for detailed specification see footnote 26; observations from treatments 24R and 24R+A pooled; N=576.

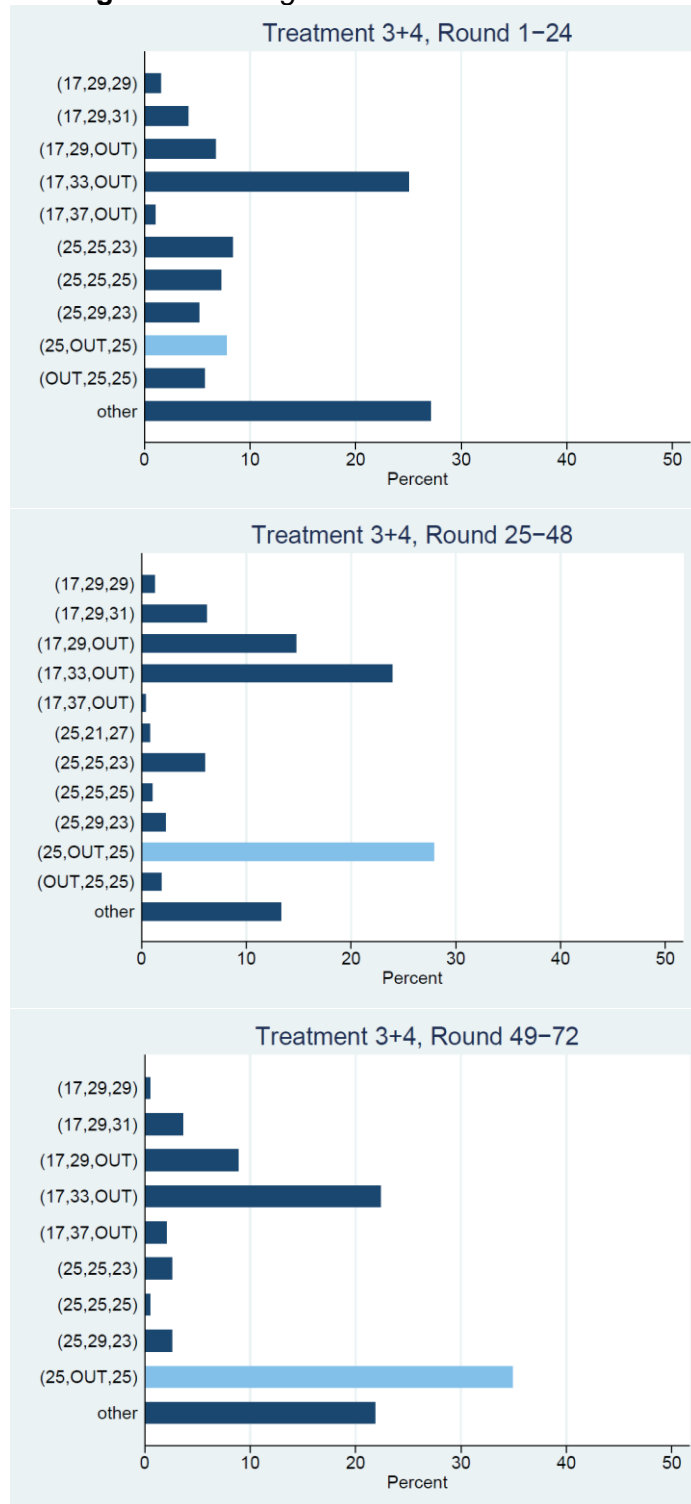
With time, more learning might be possible, so an increase in rounds appears to be the next reasonable step. New subjects were employed, and three sessions were conducted with a length of 48 rounds and 2 sessions with a length of 72 rounds.^{27,28}

Figure 3 shows results for the last two treatments, where we first see that the results for the first 24 rounds are similar to those of the first two treatments. In rounds 25 – 48, however, behavior changes substantially: The most frequent observation is the unique SPNE outcome with about 28%, and this number increases to 35% for rounds 49-72 in treatment 72R, while $(17, 33, OUT)$ is only the second most frequent observation.

²⁷ For rounds 1-24 and 25-48, 480 observations were collected from treatments 48R and 72R, while 192 observations were collected for rounds 49-72 from treatment 72R.

²⁸ From here on out the data from treatments 48R and 72R is pooled for rounds 1 – 48 because I detect no significant differences in any variables between these treatments in rounds 1 – 24 and in rounds 25 – 48; see Tables 7 and 8 in appendix A.4.

Figure 3 *Learning in Treatments 48R and 72R*



Notes: Locational choice triples on the y-axis, frequency in percent on the x-axis; triples with less than 20 observations are merged in “other”; unique SPNE outcome highlighted; N=480 for rounds 1-48 and N=192 for rounds 49-72.

Concerning differences over time, I find that in rounds 25 – 48 the unique SPNE outcome is played significantly more often ($p = 0.049$, OLS regression, see footnote 26 for specification) than in rounds 1 – 24. In rounds 49 – 72, there is again a rise in (25, *OUT*, 25) compared to rounds 25 – 48, but the rise is only weakly significant ($p = 0.055$, OLS regression, see footnote 26 for specification); see Table 9 in appendix A.4 for both regressions. The frequency of SPNE play also rises over time, as we can see in Table 3, but the most substantial change seems to occur in rounds 25 – 48; this will be explored further in the next section.

Table 3: *Learning over time*

SPNE play by	Rounds 1-24	Rounds 25-48	Rounds 49-72
Player 1	28%	41%	44%
Player 2	23%	43%	49%
Player 3	63%	84%	84%

Notes: All treatments pooled; $N=1056$ for rounds 1-24, $N= 480$ for rounds 25-48, $N= 192$ for rounds 49-72.

3.4 A closer Look at Learning

With the information gained in Sections 3.1 to 3.3, as well as utilizing insights gained from the questionnaires, there is evidence that learning to play the SPNE outcome is a multi-step process in this game, which can be observed across all sessions at variable speeds. I will explain these stages in detail in this section, and show which steps lead from the players' first play of (17, 33, *OUT*) to the unique SPNE outcome as the most frequent observation in the end.²⁹

Step 1. Player 1 and 2 corner the market: (17, 33, *OUT*)

As we have seen in all treatments in Sections 3.1, 3.2 and 3.3, in early rounds of the game

²⁹ For the following analysis, I pool the data from all treatments, as OLS regressions show no significant differences across treatments; see Section A.4 in the appendix.

(17, 33, OUT) is the most frequent observation. Intuitively, this means that the players in position 1 and 2 share the win and make it impossible for player 3 to enter the game profitably, thereby “cornering the market”. In the first 24 rounds, 59% of players in position 1 choose $a_1 = 17$, and 55% of players in position 2 respond by playing $a_2 = 33$ given $a_1 = 17$ (which is not a best response according to the SPNE), while players in position 3 make payoff-maximizing choices in a majority of cases (65%). Across all sessions, (17, 33, OUT) is played 29% of the time in rounds 1-24.

We learn from the questionnaires of treatments 24R and 24R+A, where the number of rounds was 24 and therefore relatively low, that position 3 is seen as the most powerful and “easy to play” by nearly all of the subjects, as there is no uncertainty about other players’ behavior when player 3 makes his decision. So it perhaps comes as no surprise that players in position 3 choose the payoff-maximizing $a_3 = OUT$ after $a_1 = 17$ and $a_2(a_1 = 17) = 33$ as their most frequent action. Additionally, choosing $a_2 = 33$ after $a_1 = 17$ in position 2 was believed to be the payoff-maximizing play by about half of the subjects.³⁰ Therefore, when players in position 1 learned that by playing $a_1 = 17$ player 2 would react by playing $a_2 = 33$ and player 3 would stay out of the game, they continued with this strategy.

About one third of players in position 1 do not believe entering at the median to be profitable according to the questionnaires, as they believe the other two players would respond by choosing locations close to player 1. About 20% of players in position 1 also say that they felt “lost” or “confused”, as they had to predict the following players’ behavior. And indeed, while $a_1 = 17$ is not according to the SPNE, in terms of payoff it is favorable: Over all treatments, players in position 1 who play $a_1 = 17$ earn 239% more than those who play $a_1 = 25$ in rounds 1 – 24.

To sum up, in the beginning of the game a majority of players in position 1 play $a_1 = 17$, and most players in position 2 respond by playing $a_2(a_1 = 17) = 33$. Player 3 then has an easy choice to make, as there is no profitable way for him to enter the game and win, and therefore is the only player that acts according to the SPNE. Deviations from this strategy also do not pay off in the short term for player 1, as $a_1 = 17$ is far more profitable

³⁰ When we look at the questionnaires from treatments 24R and 24R+A, roughly 12% of subjects state that they had a hard time calculating the best response to $a_1 = 17$ when they were in position 2, and 9% of subjects indicate that they were able to figure out that the best response to $a_1 = 17$ is $a_2 = 29$, but were unsure whether to make this choice as player 2 due to uncertainty about player 3’s behavior.

than $a_1 = 25$ in early rounds of the game. Players in position 2 have a hard time to calculate the best response to $a_1 = 17$, and given that they share a win with player 1 if they play $a_2(a_1 = 17) = 33$, a majority of players in position 2 choose to continue with this strategy. These considerations put together explain that $(17, 33, OUT)$ is the most frequent outcome in early rounds of the game.

Step 2. Player 2 starts to best-respond: $(17, 29, OUT)$

As we have seen in Sections 3.1 and 3.2, in rounds 13-24 the number of players in position 2 responding to $a_1 = 17$ with $a_2 = 29$ (which is indeed their best response) rises significantly, while players in position 3 still choose the payoff-maximizing option of $a_3 = OUT$ a majority of the time (65%) given $a_2(a_1 = 17) = 29$. The payoff of players in position 2 consequently rises in rounds 13-24 compared to rounds 1-12 by 20.2% ($p = 0.038$)³¹, as player 2 more often wins alone rather than share the win with player 1.

Step 3. Player 1 chooses the median: $(25, a_2, a_3)$:

As players in position 1 start to lose more frequently given that player 2 best responds more often, a significant rise of $a_1 = 25$ can be observed after round 24 ($p = 0.022$).³² What triggers this change? A location closer to the edge, i.e. $a_1 < 17$, is almost never chosen by player 1 (which is correct, as players that do choose these locations win less than 10% of cases). If a player deviates from $a_1 = 17$, some players (10% in rounds 13 – 48) choose $a_1 = OUT$ and stay out of the game completely, thinking that there is no possible location to win. However, most deviations from $a_1 = 17$ occur to $a_1 = 25$ (84% in rounds 13 – 48), which is in accordance with the SPNE.

These deviations from $a_1 = 17$ occur even though it is still more profitable for player 1 to play $a_1 = 17$ compared to $a_1 = 25$ for most of the game (by 177% in rounds 13 – 24, and by 48% in rounds 25 – 48). However, in rounds 49-72, enough players in position 2 best respond, so it gets more profitable for player 1 to play $a_1 = 25$ over $a_1 = 17$ by 8%.

³¹ OLS regression on the individual level clustered by session across all treatments. Dependent variable is the mean payoff when in position 2, independent variable is a dummy for rounds 1-12 vs. 13-24; see footnote 26 for detailed specification.

³² OLS regression on the individual level clustered by session. Dependent variable is the frequency of choosing $a_1 = 25$ when in position 1, independent variable is a dummy for rounds 25-72 vs. 1-24; see footnote 26 for detailed specification.

Step 4. The unique SPNE outcome is the most frequent observation: (25, OUT, 25)

As player 1 shifts his behavior towards $a_1 = 25$, the unique SPNE outcome is not played right away. In the first 24 rounds, only 22% of players in position 2 play $a_2 = OUT$ after $a_1 = 25$, which would be according to the SPNE. This number rises significantly to 67% in rounds 25–48 and to 85% in rounds 49 – 72 ($p = .043$, OLS regression with a dummy for rounds 25 – 48 vs. 1 – 24, see footnote 26 for specification).

After the early stages of the game, however, as we have seen in Section 3.3, (25, OUT, 25) is the most frequent observation overall in rounds 25 – 72, and the rise in plays of the unique SPNE outcome is significant compared to rounds 1 – 24.

3.5 Summary of Results

I will now summarize why players converge towards the unique SPNE outcome, given that initial play consists mainly of (17, 33, OUT). All deviations from this strategy are towards the SPNE: First, player 2 best responds to $a_1 = 17$, and then as player 1 loses more frequently he deviates to the SPNE action of $a_1 = 25$ more often. As more players in position 1 play $a_1 = 25$, players in position 2 also learn over time to best respond, and when they do, the unique SPNE outcome can emerge as the most frequent observation.

However, it is player 2's actions that are especially crucial for the emergence of the unique SPNE outcome. Seeing that player 3 is best responding in a majority of cases, and player 1 is profit maximizing (as $a_1 = 17$ is on average far more profitable than $a_1 = 25$ given the behavior of the other players, except in very late stages of the game), it is therefore player 2's behavior that changes most. In the beginning, player 2's reluctance to best respond to $a_1 = 17$ makes player 1's deviation from the SPNE profitable. In rounds 13 – 24 player 2 starts to best respond more often. Given that player 3 mostly best responds to any (a_1, a_2) , and even more so in later stages of the game, after players in position 1 start to play $a_1 = 25$ more frequently, it is again player

2's realization that he cannot win and should best respond to $a_1 = 25$ with $a_2 = \text{OUT}$ that drives the emergence of the unique SPNE outcome.

Table 4 shows that across all player positions and sessions, there was considerable learning: For all given variables, actions according to the SPNE increased over time, and choices that are not in accordance with the SPNE decreased. I therefore conjecture that with more time and more learning, the prevalence of the SPNE outcome would be even stronger.

Table 4: *Summary Statistics*

frequency of	Round			p-value
	1-24	25-48	49-72	
SPNE play in position 3	.65	.84	.84	.001
SPNE play in position 2	.25	.43	.49	.001
SPNE play in position 1	.30	.41	.47	.022
$a_1 = 17$.59	.54	.44	.761
$a_2 = 33$ given $a_1 = 17$.55	.51	.50	.535
$a_2 = 29$ given $a_1 = 17$.31	.43	.44	.240
$a_2 = \text{OUT}$ given $a_1 = 25$.22	.67	.85	.043
(25, OUT, 25)	.04	.28	.35	.007

Notes: Observations from all treatments pooled; p-value for the round-dummy of an OLS regression on the individual level clustered by session, the dependent variable is the frequency of the variable on the left, independent variable is a dummy for rounds 25-72 vs. 1-24; see footnote 26 for detailed specification.

4 Conclusion

In this paper, I report on a theoretical and experimental investigation of a 3-player sequential-entry variant of Hotelling's locational choice model (1929) that was proposed by Osborne and Kats. Despite clear predictions due to the uniqueness of the SPNE outcome, the experiment reveals that initial play in the experiment is not in accordance

with the SPNE. However, after many repetitions play does converge toward the unique SPNE outcome.

As was stated in the introduction, this model can also be used to describe plurality-rule elections. Even when behavior is not according to the SPNE, initial play suggests that a two-party system would emerge, so Duverger's law is robust to violations of the SPNE in this variant of Hotelling's model.

On a final note, we find that in many finitely repeated games with a unique equilibrium prediction, these predictions are systematically violated when tested in the lab or empirically, even in simpler environments than the one considered in this paper. Examples include ultimatum games (e.g. Roth et al. (1991) or Slonim and Roth (1998)), public goods games (see Ledyard (1995) and Chaudhuri (2011)) or the centipede game (e.g. McKelvey and Palfrey (1992)). Therefore, it is perhaps surprising that play converges toward the unique SPNE outcome in the sequential Hotelling game at all. In fact, with a shorter time horizon, I would have concluded that also in this complicated setting, behavior does not converge to the SPNE outcome at all, as we can never know when we are dealing with the long run, until it is here.

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A Appendix

A.1 Proof for the SPNE of the sequential Hotelling game

Lemma 1. In any SPNE outcome in the sequential Hotelling game, no players are losing.

Proof. As players can always guarantee themselves a payoff of $\pi_i = 0$ by choosing $a_i = OUT$, a player will always have an incentive to deviate if he loses. \square

Lemma 2. In any SPNE, if exactly two players enter the game, those players will enter at the median of 0.5.

Proof. We assume that exactly two players i and j enter the game. If player i enters the game at a location $a_i < 0.5$ (w.l.o.g.), player j wins if he locates at the median itself, as j gets half the votes from $[0.5, 1]$, plus $\frac{x_j - x_i}{2}$, i.e. $v = 0.5 + \frac{x_j - x_i}{2}$, which is more than half of the votes.

If both players locate at the median, they get the same number of votes, and if one player would deviate that player would get less votes and lose, which can never be part of an SPNE by Lemma 1. \square

Lemma 3. If player 1 enters the game at $a_1 < 0.5$ and player 2 plays according to the following strategy profile \widehat{a}_2 , and player 3 plays $a_3 = OUT$, then player 2 will win.

$$\widehat{a}_2 \in \begin{cases} [\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1] & \text{if } a_1 < \frac{1}{6} \\ [\frac{2-a_1}{3}, 1 - a_1) & \text{if } a_1 \geq \frac{1}{6} \end{cases}$$

Proof. With the above strategy \widehat{a}_2 , player 2 always gets more votes than player 1 because he locates closer to the median, i.e. $0.5 - a_1 > a_2 - 0.5$. \square

Theorem 1. All subgame-perfect Nash equilibria of the sequential Hotelling game are given by:

$$a_1^* = 0.5, a_2^*(a_1) = \begin{cases} 0.5 & \text{if } a_1 = OUT \\ \left[\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1\right] & \text{if } a_1 < \frac{1}{6} \\ \left[\frac{2-a_1}{3}, 1 - a_1\right) & \text{if } a_1 \geq \frac{1}{6} \text{ and } a_1 < 0.5 \\ OUT & \text{if } a_1 = 0.5, \end{cases}$$

while player 3 chooses according to the following rules:

1. If the set $A = \{a_3 | v_3 > \max(v_1, v_2)\}$ is nonempty, i.e. if player 3 can attain $v_3 > \max(v_1, v_2)$ by choosing some $a_3 \in [0, 1]$, he chooses one of these payoff-maximizing choices.
2. If set A is empty and the set $B = \{a_3 | v_3 = \max(v_1, v_2)\}$ is nonempty, i.e. player 3 can attain $v_3 = \max(v_1, v_2)$ by choosing some $a_3 \in [0, 1]$, he chooses one of them.
3. If both sets A and B are empty, $a_3 = OUT$.

Proof. As this is a backward induction problem, we start by looking at player 3's strategy. As player 3 is the last player to act, he simply goes through all possible location choices and chooses one of the payoff-maximizing choices given (a_1, a_2) . Therefore, if player 3 chooses an a_3 according to the above strategy profile, a_3 is a best response to player 1 and 2's actions.³³

As far as player 2's best responses are concerned, we have four cases, depending on the action of player 1:

Case 1: $a_1 = OUT$

Case 2: $a_1 < \frac{1}{6}$

Case 3: $a_1 \geq \frac{1}{6}$ and $a_1 < 0.5$

³³ Note as there are a great number of variations off the equilibrium path, player 3's strategy given all possible histories (a_1, a_2) is too big to explicitly write down here.

Case 4: $a_1 = 0.5$.

We will go through the different cases one by one.³⁴ Player 2 always has two goals, which he tries to fulfill in order. **Goal 1:** Anticipating player 3's best responses, player 2 first checks whether he can locate in such a way that it is not possible for player 3 to win the game, i.e. deterring player 3 from entering, while achieving $v_2 > v_1$. If goal 1 can be achieved, player 2 wins alone, which is preferable to all other outcomes. If goal 1 cannot be achieved, player 2 tries to achieve **Goal 2:** Player 2 tries to find a location such that he shares a win with the smallest number of players. Finally, if neither goal 1 nor goal 2 can be achieved by player 2, i.e. if player 2 cannot win, $a_2^* = OUT$.

Case 1: $a_1 = OUT$

In this case I show that player 2 cannot achieve goal 1, and he achieves goal 2 by playing $a_2^*(a_1 = OUT) = 0.5$.

If player 2 plays $a_2^*(a_1 = OUT) = 0.5$, player 3's best response is to also locate at the median of 0.5 and share the win with player 2 by Lemma 2. As $a_2(a_1 = OUT) \neq 0.5$ cannot be part of an SPNE by Lemma 2 and Lemma 1, and as player 2 can guarantee himself a shared win with player 3 by playing $a_2^*(a_1 = OUT) = 0.5$, it is the best response.

Case 2: $a_1 < \frac{1}{6}$

In this case I show that player 2 can achieve goal 1 by playing any $a_2^* \left(a_1 < \frac{1}{6} \right) \in \left[\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1 \right]$.

This means that player 2 wins alone by deterring player 3 from entering (i.e. $a_3 = OUT$) while achieving $v_2 > v_1$ if $a_2^* \left(a_1 < \frac{1}{6} \right) \in \left[\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1 \right]$.

The analysis of Case 2 is structured as follows: First I show that if player 2 locates out of the given best response range, player 3 would win alone; therefore, any action outside the best response range cannot be part of an SPNE by Lemma 1. Then I show that given $a_1 < \frac{1}{6}$ all actions in $\left[\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1 \right]$ lead to player 2 winning alone, thereby achieving goal 1.

³⁴ Note that due to the symmetric nature of the game, we will omit all cases of $a_1 > 0.5$ w.l.o.g.

First note that we can exclude any $a_2 \left(a_1 < \frac{1}{6} \right) < 0.5$ as best responses by player 2, as player 3 can then locate at the median and win alone.

Next, I show that any $a_2 \left(a_1 < \frac{1}{6} \right) > \frac{2}{3} + a_1$ cannot be a best response, as player 3 can then find a location a_3 with $a_1 < a_3 < a_2$ and win alone. If $a_2 = \frac{2}{3} + a_1 + \varepsilon$ where $\varepsilon > 0$ and such that $a_2 \in \left(\frac{2}{3} + a_1, 1 \right]$, the vote shares in this case are given by $v_1 = \frac{a_1 + a_3}{2}$, $v_2 = 1 - \frac{a_2 + a_3}{2}$ and $v_3 = \frac{a_2 - a_1}{2}$. For player 3 to enter, two inequalities have to be fulfilled: $v_3 > v_1$, which holds iff $a_3 > \frac{2}{3} - a_1 + \varepsilon$, and $v_3 > v_2$, which holds iff $a_3 > \frac{2}{3} - a_1 - 2\varepsilon$. As the first two terms $\frac{2}{3} - a_1$ on the right hand side are the same in both inequalities, we see that as long as ε is positive, player 3 can enter the game and win alone. Therefore, any $a_2 \left(a_1 < \frac{1}{6} \right) > \frac{2}{3} + a_1$ cannot be a best response by Lemma 1.

Now I show that $a_2 \left(a_1 < \frac{1}{6} \right) < \frac{2}{3} - \frac{a_1}{3}$ cannot be a best response. Suppose $a_2 = \frac{2 - a_1}{3} - \varepsilon$ where ε is positive and such that $a_2 \in \left(a_1, \frac{2 - a_1}{3} \right)$. I will show that player 3 can then locate at $a_3 = \frac{2}{3} - \frac{a_1}{3}$ and win alone. The vote shares in this case are given by $v_1 = \frac{a_1 + a_2}{2}$, $v_2 = \frac{a_3 - a_1}{2}$ and $v_3 = 1 - \frac{a_2 + a_3}{2}$. Again, two inequalities have to be fulfilled for player 3 to enter and win: $v_3 > v_1$ and $v_3 > v_2$. $v_3 > v_1$ holds iff $a_3 < \frac{2 - a_1}{3} + 2\varepsilon$, which simplifies to $0 < 2\varepsilon$ by plugging in $a_3 = \frac{2}{3} - \frac{a_1}{3}$. $v_3 > v_2$ holds iff $a_3 < 1 + \frac{a_1 - a_2}{2}$, which simplifies to $a_1 > -\frac{\varepsilon}{2}$ by plugging in a_2 and a_3 ; both inequalities are always fulfilled. As I have shown that if $a_2 \left(a_1 < \frac{1}{6} \right) = \frac{2}{3} - \frac{a_1}{3} - \varepsilon$ player 3 can enter at $a_3 = \frac{2}{3} - \frac{a_1}{3}$ and win alone, any $a_2 \left(a_1 < \frac{1}{6} \right) < \frac{2}{3} - \frac{a_1}{3}$ cannot be a best response by Lemma 1.

Now we have established that for all location choices by player 2 outside of $\left[\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1 \right]$ given $a_1 < \frac{1}{6}$, player 3 can find a location to win alone, so these actions cannot be best responses for player 2 by Lemma 1. I proceed to show that for all location choices $a_2^* \left(a_1 < \frac{1}{6} \right) \in \left[\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1 \right]$, player 3 will play $a_3 = OUT$. It then follows from Lemma 3 that $a_2^* \left(a_1 < \frac{1}{6} \right) \in \left[\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1 \right]$ is the best response correspondence for player

2 if $a_1 < \frac{1}{6}$. I show this by going through all possible location choices $a_3 \in [0, 1]$ for player 3 (Cases 2a-2e) and showing for each case that player 3 cannot win because either $v_3 > v_2$ or $v_3 > v_1$ cannot be fulfilled.³⁵ It then follows that player 3's best response given the history $a_1 < \frac{1}{6}, a_2 \in \left[\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1\right]$, is $a_3 = OUT$, as player 3 cannot win if he chooses any action other than $a_3 = OUT$.

(Case 2a) $a_3 < a_1$: To show that player 3 will not enter at $a_3 < a_1$, we have to show that player 3 loses if he enters at any a_3 with $a_3 < a_1$ given the history $a_1 < \frac{1}{6}, a_2 \in \left[\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1\right]$. In this case the vote shares are given by $v_1 = \frac{a_2 - a_3}{2}$, $v_2 = 1 - \frac{a_1 + a_2}{2}$ and $v_3 = \frac{a_1 + a_2}{2}$, and therefore $v_3 > v_2$ iff $a_3 > 2 - 2a_1 - a_2$. Given player 2's best response range $a_2^* \left(a_1 < \frac{1}{6}\right) \in \left[\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1\right]$, $a_3 > 2 - 2a_1 - a_2$ is easiest to fulfill at the lower bound³⁶ () for $a_2^* \left(a_1 < \frac{1}{6}\right)$. Therefore, if the inequality cannot be satisfied for the case of the lower bound of a_2^* , it cannot be satisfied for the whole best response range. If we plug in the lower bound, $v_3 > v_2$ iff $a_3 > \frac{4}{3} - \frac{5a_1}{3}$. As a_1 is bounded from above by $\frac{1}{6}$, if we were to plug in $a_1 = \frac{1}{6}$, then $v_3 > v_2$ iff $a_3 > \frac{19}{18}$, which can never be satisfied, so player 3 will not enter in case 2a.

(Case 2b) $a_3 = a_1$: To show that player 3 will not enter at $a_3 = a_1$, we must show that player 3 loses if he enters at $a_3 = a_1$ given the history $(a_1 < \frac{1}{6}, a_2 \in \left[\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1\right])$. In this case $v_1 = v_3 = \frac{a_1 + a_2}{4}$ and $v_2 = 1 - \frac{a_1 + a_2}{2}$, and $v_3 > v_2$ iff $a_3 > \frac{4}{3} - a_2$. Given player 2's best response range, this inequality is easiest to fulfill at the upper bound for $a_2^* \left(a_1 < \frac{1}{6}\right)$. If we plug in the upper bound, $v_3 > v_2$ iff $a_3 > \frac{1}{3}$, which can never be true as $a_3 = a_1 < \frac{1}{6}$, so player 3 cannot win in case 2b.

(Case 2c) $a_1 < a_3 < a_2$: To show that player 3 will not choose any a_3 with $a_1 < a_3 < a_2$,

³⁵ Note that in cases 2a-2e we have to derive player 3's best responses because we did not explicitly write down player 3's strategy given all possible histories.

³⁶ From here on out, "easiest to fulfill at the lower/upper bound" means that if the inequality is not fulfilled at the lower/upper bound, it cannot be fulfilled at all.

we have to show that player 3 loses if he enters at any a_3 with $a_1 < a_3 < a_2$ given the history $(a_1 < \frac{1}{6}, a_2 \in [\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1])$. In this case, the vote shares are given by $v_1 = \frac{a_1 + a_3}{3}$, $v_2 = 1 - \frac{a_2 + a_3}{2}$ and $v_3 = \frac{a_2 - a_1}{2}$. For player 3 to enter, two inequalities have to be fulfilled. $v_3 > v_1$ holds iff $2a_1 < a_2 - a_3$ and $v_3 > v_2$ holds iff $a_1 > 2a_2 + a_3 - 2$. To see that both of these inequalities cannot be fulfilled at the same time, suppose $a_2 = \frac{2}{3} + a_1 - \varepsilon$, where $\varepsilon > 0$ and such that $a_2 \in [\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1]$. The two inequalities then simplify to $a_3 < \frac{2}{3} - a_1 - \varepsilon$ and $a_3 > \frac{2}{3} - a_1 + 2\varepsilon$. We see that, as the first two terms $\frac{2}{3} - a_1$ on the right hand side are the same, as long as $\varepsilon > 0$, both inequalities cannot be fulfilled at the same time. It follows that player 3 cannot win in case 2c.

(Case 2d) $a_3 = a_2$: To show that player 3 will not enter at $a_3 = a_2$, we have to show that player 3 loses if he enters at $a_3 = a_2$ given the history $(a_1 < \frac{1}{6}, a_2 \in [\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1])$. In this case the vote shares are given by $v_1 = \frac{a_1 + a_2}{2}$, $v_2 = \frac{1}{2} - \frac{a_1 + a_2}{4}$, and $v_3 > v_1$ holds iff $a_2 < \frac{2}{3} - a_1$. This inequality can never be fulfilled as $a_1 < \frac{1}{6}$ and player 2's best response range is bounded from below by $\frac{2}{3} - \frac{a_1}{3}$, so player 3 cannot win in case 2d.

(Case 2e) $a_3 > a_2$: To show that player 3 will not choose any $a_3 > a_2$, we must show that player 3 loses if he enters at any $a_3 > a_2$ given the history $(a_1 < \frac{1}{6}, a_2 \in [\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1])$. In this case the vote shares are given by $v_1 = \frac{a_1 + a_2}{2}$, $v_2 = \frac{a_3 - a_1}{2}$ and $v_3 = 1 - \frac{a_2 + a_3}{2}$, and therefore $v_3 > v_2$ iff $a_3 < 1 + \frac{a_1 - a_2}{2}$. As by assumption $a_3 > a_2$, if the

right hand side of inequality $a_3 < 1 + \frac{a_1 - a_2}{2}$ should equal a_2 , player 3 cannot find a location such that $v_3 > v_2$ is fulfilled. So, if we solve the equation $a_2 = 1 + \frac{a_1 - a_2}{2}$ for a_2 , we get the lower bound for player 2's best response range, $a_2 = \frac{2}{3} - \frac{a_1}{3}$. As a_2 is subtracted in the inequality $a_3 < 1 + \frac{a_1 - a_2}{2}$, if $a_2 < \frac{2}{3} - \frac{a_1}{3}$, player 3 can find a location such that both $a_3 > a_2$ and $a_2 < 1 + \frac{a_1 - a_2}{2}$ are fulfilled. However, if $a_2 \geq \frac{2}{3} - \frac{a_1}{3}$, player 3 cannot find a location such that both $a_3 > a_2$ and $a_2 < 1 + \frac{a_1 - a_2}{2}$ are fulfilled. Therefore, player 3

will not enter the game at an a_3 with $a_3 > a_2$ after the history $(a_1 < \frac{1}{6}, a_2 \in [\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1])$.

To sum up, player 2 loses if he plays any $a_2^*(a_1 < \frac{1}{6}, a_2 \notin [\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1])$, and player 2 can deter player 3 from entering the game if he plays any $a_2^*(a_1 < \frac{1}{6}, a_2 \in [\frac{2}{3} - \frac{a_1}{3}, \frac{2}{3} + a_1])$. Player 2 therefore wins alone with this best response range by Lemma 3, fulfilling Goal 1.

Case 3: $\frac{1}{6} \leq a_1 < 0.5$

In this case I show that player 2 can achieve goal 1 by playing any $a_2^*(\frac{1}{6} \leq a_1 < 0.5) \in [\frac{2-a_1}{3}, 1-a_1]$.³⁷ This means that player 2 wins alone by deterring player 3 from entering while achieving $v_2 > v_1$ if $a_2^*(\frac{1}{6} \leq a_1 < 0.5) \in [\frac{2-a_1}{3}, 1-a_1]$. The analysis of case 3 is structured similarly to case 2: First I show that if player 2 locates out of the given best response range, player 2 will either lose or tie for the win with player 1. Then I show that given $\frac{1}{6} \leq a_1 < 0.5$ all actions in $[\frac{2-a_1}{3}, 1-a_1]$ lead to player 2 winning alone, thereby achieving goal 1.

First note that we can exclude any $a_2(\frac{1}{6} \leq a_1 < 0.5) < \frac{2-a_1}{3}$ as best responses by player 2, as player 3 can then locate at the median and win alone.

Next, I show that any $a_2(\frac{1}{6} \leq a_1 < 0.5) \geq 1-a_1$ cannot be a best response. This stems from the fact that even if player 3 does not enter at all, player 2 will split the win with player 3 (in case of $a_2(\frac{1}{6} \leq a_1 < 0.5) = 1-a_1$) or lose (in case of $a_2(\frac{1}{6} \leq a_1 < 0.5) > 1-a_1$), as player 1 would then be located as close or closer to the median as player 2, thereby gaining the same number of votes or more votes than player 2. This

³⁷ Again, I omit the case of $a_1 > 0.5$ due to symmetry w.l.o.g.

would in the best case fulfill goal 2 for player 2 (splitting the win with player 1), but as we are about to see, player 2 can fulfill goal 1 by playing any $a_2^* \left(\frac{1}{6} \leq a_1 < 0.5 \right) \in \left[\frac{2-a_1}{3}, 1-a_1 \right)$, so any $a_2 \left(\frac{1}{6} \leq a_1 < 0.5 \right) \geq 1-a_1$ cannot be a best response given $\frac{1}{6} \leq a_1 < 0.5$.

Now I show that $a_2 \left(\frac{1}{6} \leq a_1 < 0.5 \right) < \frac{2-a_1}{3}$ cannot be a best response. Suppose $\frac{1}{6} \leq a_1 < 0.5$ and $a_2 = \frac{2-a_1}{3} - \varepsilon$ where ε is positive and such that $a_2 \in \left(a_1, \frac{2-a_1}{3} \right)$. I will show that player 3 can then locate at $a_3 = \frac{2}{3} - \frac{a_1}{3}$ and win alone. The vote shares in this case are given by $v_1 = \frac{a_1+a_2}{2}$, $v_2 = \frac{a_3-a_1}{2}$ and $v_3 = 1 - \frac{a_2+a_3}{2}$. Two inequalities have to be fulfilled for player 3 to enter and win: $v_3 > v_1$ and $v_3 > v_2$. $v_3 > v_2$ holds iff $a_3 < 1 + \frac{a_1-a_2}{2}$, which simplifies to $a_1 > -\frac{\varepsilon}{2}$ by plugging in a_2 and a_3 , which is always satisfied. $v_3 > v_1$ holds iff $a_3 > -2 + a_1 + 2a_2$, which simplifies to $0 < \varepsilon$ by plugging in a_2 and a_3 , which is also always satisfied. As I have shown that if $a_2 \left(\frac{1}{6} \leq a_1 < 0.5 \right) = \frac{2-a_1}{3} - \varepsilon$ player 3 can enter at $a_3 = \frac{2}{3} - \frac{a_1}{3}$ and win alone, any $a_2 \left(\frac{1}{6} \leq a_1 < 0.5 \right) < \frac{2-a_1}{3}$ cannot be a best response by Lemma 1.

Now we have established that for all location choices by player 2 outside of $\left[\frac{2-a_1}{3}, 1-a_1 \right)$ given $\frac{1}{6} \leq a_1 < 0.5$, player 2 will lose or split the win with player 1, so these actions cannot be best responses for player 2 as he can achieve goal 1. I proceed to show that for all location choices $a_2^* \left(\frac{1}{6} \leq a_1 < 0.5 \right) \in \left[\frac{2-a_1}{3}, 1-a_1 \right)$, player 3 will play $a_3 = OUT$. It then follows from Lemma 3 that $a_2^* \left(\frac{1}{6} \leq a_1 < 0.5 \right) \in \left[\frac{2-a_1}{3}, 1-a_1 \right)$ is the best response correspondence for player 2 if $\frac{1}{6} \leq a_1 < 0.5$ because player 2 wins alone and thereby fulfills goal 1. I show this by going through all possible location choices $a_3 \in [0,1]$ for player 3 (Cases 3a-3e) and showing for each case that player 3 cannot win because either $v_3 > v_2$ or $v_3 > v_1$ cannot be fulfilled.³⁸ It then follows that player 3's

³⁸ Note that in cases 2a-2e we have to derive player 3's best responses because we did not explicitly write down player 3's strategy given all possible histories.

best response given the history $(\frac{1}{6} \leq a_1 < 0.5, a_2 \in [\frac{2-a_1}{3}, 1-a_1])$ is $a_3 = OUT$, as player 3 cannot win if he chooses any action other than $a_3 = OUT$.

(Case 3a) $a_3 < a_1$: To show that player 3 will not enter at $a_3 < a_1$, we have to show that player 3 loses if he enters at any a_3 with $a_3 < a_1$ given the history $(\frac{1}{6} \leq a_1 < 0.5, a_2 \in [\frac{2-a_1}{3}, 1-a_1])$. In this case the vote shares are given by $v_1 = \frac{a_2-a_3}{2}$, $v_2 = 1 - \frac{a_1+a_2}{2}$ and $v_3 = \frac{a_1+a_3}{2}$, and therefore $v_3 > v_2$ iff $a_3 > 2 - 2a_1 - a_2$. Given player 2's best response range $a_2^* (\frac{1}{6} \leq a_1 < 0.5) \in [\frac{2-a_1}{3}, 1-a_1]$, $a_3 > 2 - 2a_1 - a_2$ is easiest to fulfill at the lower bound for $a_2^* (a_1 < \frac{1}{6})$. If we plug in the lower bound, $v_3 > v_2$ iff $a_3 > \frac{4}{3} - \frac{5a_1}{3}$. As a_1 is bounded from above by 0.5, if we were to plug in $a_1 = 0.5$, then $v_3 > v_2$ iff $a_3 > \frac{1}{2}$, which of course can never be satisfied as by assumption $a_3 < a_1$, so player 3 will not enter in case 3a.

(Case 3b) $a_3 = a_1$: To show that player 3 will not enter at $a_3 = a_1$, we have to show that player 3 loses if he enters at $a_3 = a_1$ given the history $(\frac{1}{6} \leq a_1 < 0.5, a_2 \in [\frac{2-a_1}{3}, 1-a_1])$. In this case $v_1 = v_3 = \frac{a_1+a_2}{4}$ and $v_2 = 1 - \frac{a_1+a_2}{2}$, and $v_3 > v_2$ iff $a_1 > \frac{4}{3} - a_2$. Suppose $a_2 = 1 - a_1 - \varepsilon$, where $\varepsilon > 0$ and such that $a_2 \in [\frac{2-a_1}{3}, 1-a_1]$. $a_1 > \frac{4}{3} - a_2$ then simplifies to $0 > \frac{1}{3} + \varepsilon$, which can never be true, so player 3 cannot win in case 3b.

(Case 3c) $a_1 < a_3 < a_2$: To show that player 3 will not choose any a_3 with $a_1 < a_3 < a_2$, we have to show that player 3 loses if he enters at any a_3 with $a_1 < a_3 < a_2$ given the history $(\frac{1}{6} \leq a_1 < 0.5, a_2 \in [\frac{2-a_1}{3}, 1-a_1])$. In this case, the vote shares are given by $v_1 = \frac{a_1+a_3}{2}$, $v_2 = 1 - \frac{a_2+a_3}{2}$ and $v_3 = \frac{a_2-a_1}{2}$. For player 3 to enter, two inequalities have to be fulfilled: $v_3 > v_1$ holds iff $2a_1 < a_2 - a_3$ and $v_3 > v_2$ holds iff $a_1 < 2a_2 + a_3 - 2$. To see that both of these inequalities cannot be fulfilled at the same time, suppose $a_2 = 1 - a_1 - \varepsilon$, where $\varepsilon > 0$ and such that $a_2 \in [\frac{2-a_1}{3}, 1-a_1]$. The two inequalities then

simplify to $a_3 < 1 - 3a_1 - \varepsilon$ and $a_3 > 3a_1 + 2\varepsilon$. Both of these inequalities are easiest to fulfill for $a_1 = \frac{1}{6}$. If we plug in $a_1 = \frac{1}{6}$, we get $a_3 < \frac{1}{2} - \varepsilon$ and $a_3 > \frac{1}{2} + 2\varepsilon$. We see that as long as ε is positive, the inequalities cannot be fulfilled at the same time, so it follows that player 3 cannot win in case 3c.

(Case 3d) $a_3 = a_2$: To show that player 3 will not enter at $a_3 = a_2$, we have to show that player 3 loses if he enters at $a_3 = a_2$ given the history $(\frac{1}{6} \leq a_1 < 0.5, a_2 \in [\frac{2-a_1}{3}, 1 - a_1])$.

In this case the vote shares are given by $v_1 = \frac{a_1+a_2}{2}$ and $v_2 = v_3 = \frac{1}{2} - \frac{a_1+a_2}{4}$, and $v_3 > v_1$ holds iff $a_2 < \frac{2}{3} - a_1$. Given $\frac{1}{6} \leq a_1 < 0.5$, this inequality is easiest to fulfill at the lower bound of a_1 . Plugging in $a_1 = \frac{1}{6}$, $a_2 < \frac{2}{3} - a_1$ simplifies to $a_2 < \frac{1}{2}$, which can never be fulfilled as $a_3 = a_2$ and a_3 cannot be lower than 0.5 within the given response range $a_2 \in [\frac{2-a_1}{3}, 1 - a_1]$. Therefore, player 3 cannot win in case 2d.

(Case 3e) $a_3 > a_2$: To show that player 3 will not choose any a_3 with $a_3 > a_2$, we must

show that player 3 loses if he enters at any a_3 with $a_3 > a_2$ given the history $(\frac{1}{6} \leq a_1 < 0.5, a_2 \in [\frac{2-a_1}{3}, 1-a_1])$. In this case, the vote shares are given by $v_1 = \frac{a_1+a_2}{2}$, $v_2 = \frac{a_3-a_1}{2}$ and $v_3 = 1 - \frac{a_2+a_3}{2}$. For player 3 to enter, $v_3 > v_1$ must be fulfilled, which holds iff $a_3 < 2 - a_1 - 2a_2$. To see that this inequality cannot be fulfilled, suppose $a_2 = \frac{2}{3} - \frac{a_1}{3} + \varepsilon$, where $\varepsilon > 0$ and such that $a_2 \in [\frac{2-a_1}{3}, 1-a_1]$. When we plug in $a_2 = \frac{2}{3} - \frac{a_1}{3} + \varepsilon$, $a_3 < 2 - a_1 - 2a_2$ simplifies to $a_3 < \frac{2}{3} - \frac{a_1}{3} - 2\varepsilon$. We see that this inequality can never be fulfilled as long as ε is positive because by assumption $a_3 > a_2$ and a_3 would have to be lower than the lower bound of $[\frac{2-a_1}{3}, 1-a_1]$, so it follows that player 3 cannot win in case 3e.

To sum up, player 2 loses or ties for the win if he plays any $(\frac{1}{6} \leq a_1 < 0.5, a_2 \notin [\frac{2-a_1}{3}, 1-a_1])$, and player 2 can deter player 3 from entering the game if he plays any $a_2^* \in [\frac{2-a_1}{3}, 1-a_1]$. Player 2 therefore wins alone with this best response range by Lemma 3, fulfilling goal 1.

Case 4: $a_1 = 0.5$

In this case I show that player 2 can fulfill neither goal 1 nor goal 2 by choosing any $a_2 \in [0,1]$ so $a_2^*(a_1 = 0.5) = OUT$ is the best response. First, if $a_2 < 0.5$ (which also covers the case of $a_2 > 0.5$ w.l.o.g. because of the symmetric nature of the game) given $a_1 = 0.5$, player 3 can play an a_3 such that $0.5 - a_2 > a_3 - 0.5$ (i.e. he adopts a location closer to the middle) and win alone, so any $a_2 \neq 0.5$ cannot be part of an SPNE by Lemma 1. Second, if player 2 chooses $a_2(a_1 = 0.5) = 0.5$, player 3 can choose a location close to the median and win alone. Therefore, player 2 cannot win by playing any $a_2 \in [0,1]$ given $a_1 = 0.5$, so $a_2^*(a_1 = 0.5) = OUT$ is the best response.

Putting it all together

Finally, we derive player 1's action in an SPNE by looking at the outcomes for player 1 in cases 1-4. In case 1, player 1 chooses $a_1 = OUT$, which is preferable to the outcomes in cases 2 and 3, where player 3 does not enter and $v_2 > v_1$ by Lemma 3, so player 1 loses. However, if $a_1 = 0.5$ (case 4), player 2's best response $a_2^*(a_1 = 0.5) = OUT$. Player 3 will play $a_3 = 0.5$ given the history $(a_1 = 0.5, a_2^*(a_1 = 0.5) = OUT)$ by Lemma 2 in an SPNE, resulting in a shared win of players 1 and 3 in case 4. As a shared win with player 3 is preferable to $a_1 = OUT$, player 1's optimal action in an SPNE is $a_1^* = 0.5$.

Corollary 1. The unique subgame-perfect Nash equilibrium outcome for the sequential Hotelling game is given by $\{a_1 = 0.5, a_2 = OUT, a_3 = 0.5\}$.

Proof. By Theorem 1, we know that $a_1^* = 0.5$ and $a_2^*(a_1^*) = OUT$, and it follows from Lemma 2 that $a_3^*(a_1^*, a_2^*) = 0.5$. As all actions according to the SPNE are unique on the equilibrium path, the SPNE outcome is therefore also unique.

Corollary 2. Due to the symmetry of the game, assuming $a_1 \geq 0.5$, the SPNE for the sequential Hotelling game can also be written as

$$a_1^* = 0.5, \quad a_2^*(a_1) = \begin{cases} 0.5 & \text{if } a_1 = OUT \\ [\frac{1}{3} - a_1, \frac{1}{3} + \frac{a_1}{3}] & \text{if } a_1 > \frac{5}{6} \\ (a_1, \frac{1+a_1}{3}] & \text{if } a_1 \leq \frac{5}{6} \cap a_1 > 0.5 \\ OUT & \text{if } a_1 = 0.5 \end{cases}$$

while player 3 chooses according to the following rules:

1. If the set $A = \{a_3 | v_3 > \max(v_1, v_2)\}$ is nonempty, i.e. if player 3 can attain $v_3 > \max(v_1, v_2)$ by choosing some $a_3 \in [0,1]$ he chooses one of these payoff-maximizing choices.
2. If set A is empty and the set $B = \{a_3 | v_3 = \max(v_1, v_2)\}$ is nonempty, i.e. player 3 can attain $v_3 = \max(v_1, v_2)$ by choosing some $a_3 \in [0,1]$, he chooses one of them.

3. If both sets A and B are empty, $a_3 = OUT$.

Proof. In Theorem 1, we assumed w.l.o.g. that $a_1 \leq 0.5$ because of the symmetry of the game around the median of 0.5. The SPNE can also be rewritten as above while assuming $a_1 \geq 0.5$ and the proof for Theorem 1 is then equivalent if we substitute for any a_i its symmetric value $1 - a_i$.³⁹

³⁹ I define $1 - OUT = OUT$.

A.2 The lab game

The implementation of the sequential Hotelling game in the lab necessarily uses a discrete “voter base” (i.e. the locations 1 to 49), and not a continuous one. Therefore, the action spaces of all players are drastically smaller, and can be represented in a single table. In this section I will describe this Table 5 in detail, and show that the SPNE in the lab game results in the same unique SPNE outcome qualitatively, i.e. the first and the last player enter at the median, and the second player opts out.

In Table 5 we see all possible actions a_1 by player 1 in the first column, and in column 2 we see all possible actions by player 2 given all actions by player 1, i.e. all $a_2(a_1)$.⁴⁰ In column 3 of Table 5 we see all best response correspondences by player 3 given all possible histories of a_1 and a_2 , i.e. $a_3^*(a_1, a_2)$. Finally, the last column indicates winning positions for a specific sequence of moves.⁴¹

In Table 5 we can therefore represent the complete action spaces by players 1 and 2, while player 3’s strategies $a_3^*(a_1, a_2)$ in the table are best responses to all possible histories. Best responses to player 1’s actions (while accounting for player 3’s best responses) by player 2 are bold, and player 1’s payoff-maximizing choice of $a_1^* = 25$ is also bold.

As an example, if $a_1 = OUT$, player 2 in principle has three options of choosing $a_2(a_1 = OUT)$: $a_2(a_1 = OUT) = OUT$, $a_2(a_1 = OUT) \in [1, 5, \dots, 17, 21]$ or $a_2(a_1 = OUT) = 25$. If he chooses either $a_2(a_1 = OUT) = OUT$ or $a_2(a_1 = OUT) < 25$, player 3 plays one of his best responses a_3^* given (a_1, a_2) and wins alone (as indicated by the last column). As player 2 can split the win with player 3 by playing $a_2(a_1 = OUT) = 25$, that is his best response to $a_1 = OUT$ as it maximizes his payoff in the subgame following $a_1 = OUT$.

Player 2 chooses his best responses by looking at all possible outcomes in the subgame following a specific action a_1 , anticipating player 3’s best responses. From these outcomes

⁴⁰ Note that as in the sequential Hotelling game, we omit cases of $a_1 > 25$ due to the symmetry of the game around the median. Furthermore, we omit cases $a_2 > 25$ given $a_1 = OUT$ or $a_1 = 25$, as with these histories of player 1’s action, symmetry is still preserved. See footnote 16 for a more detailed explanation.

⁴¹ As the calculations for player 3’s reaction correspondences found in Table 5 are simple and lengthy, the calculations are skipped here, but are available from the author upon request.

(shown in the last column) he chooses the most favorable, i.e. he first chooses an outcome where he wins alone ("2" in the last column), then an outcome where he shares the win with another player (e.g. "2 & 3"), and if no such choices are available, he chooses $a_2 = OUT$ as his best response.

Similarly for player 1, as we assume that he anticipates the best responses by players 2 and 3 following his actions in any SPNE, player 1 knows that if he chooses $a_1 = OUT$ players 2 and 3 will win, if he chooses $a_1 < 25$ player 2 will win, and if he chooses $a_1 = 25$ he will split the win with player 3, so $a_1 = 25$ is his payoff-maximizing choice and therefore the only possibility for a_1^* in an SPNE.

Table 5: SPNE for the lab case

a_1	a_2 given a_1	a_3^* given $\{a_1, a_2\}$	Winning Position(s)
<i>OUT</i>	<i>OUT</i>	$a_3 \in X_3 \setminus OUT$	3
	$\{1, \dots, 21\}$	$a_3 \in [a_2 + 2, \dots, 48 - a_2]$	3
	25	25	2 & 3
1	<i>OUT</i>	$a_3 \in [3, \dots, 47]$	3
	1	$a_3 \in [3, \dots, 49]$	3
	$\{5, \dots, 29\}$	$a_3 \in [a_2 + 2, \dots, 47.5 - \frac{a_2}{2}]$	3
	33	33	1 & 2 & 3
	37	$a_3 \in [27, \dots, 35]$	3
	$\{41, \dots, 49\}$	$a_3 \in [1 + 2(50 - a_2), \dots, a_2 - 2]$	3
9	<i>OUT</i>	$a_3 \in [11, \dots, 39]$	3
	$\{1, 5\}$	$a_3 \in [11, \dots, 42.5 + \frac{a_2}{2}]$	3
	9	$a_3 \in [11, \dots, 49]$	3
	$\{13, \dots, 25\}$	$a_3 \in [a_2 + 2, \dots, 51.5 - \frac{a_2}{2}]$	3
	29	31	3
	{33,37}	<i>OUT</i>	2
	41	25	1 & 2 & 3
	45	$a_3 \in [19, \dots, 27]$	3
	49	$a_3 \in [11, \dots, 31]$	3
17	<i>OUT</i>	$a_3 \in [19, \dots, 31]$	3
	$\{1, \dots, 17\}$	$a_3 \in [19, \dots, 38.5 + \frac{a_2}{2}]$	3
	21	$a_3 \in [23, \dots, 39]$	3
	25	$a_3 \in [27, \dots, 31]$	3
	29	<i>OUT</i>	2
	33	<i>OUT</i>	1 & 2
	$\{37, \dots, 45\}$	<i>OUT</i>	1
	49	17	1 & 2 & 3
25	OUT	25	1 & 3
	$\{1, \dots, 9\}$	$a_3 \in [27, \dots, 34.5 + \frac{a_2}{2}]$	3
	$\{13, \dots, 21\}$	$a_3 \in [27, \dots, 48 - a_2]$	3
	25	$a_3 \in [11, \dots, 39] \setminus \{25\}$	3

Notes: This table provides the complete action space for players 1 and 2 in the lab game, as well as best responses given all histories for player 3; best responses bold for players 1 and 2; Section A.2 describes this table in detail; note that for all sets given here, $a_3 \in X_3$ and $a_2 \in X_2$ must still be satisfied.

Implications of Table 5

It follows from Table 5 that the SPNE in the lab game is characterized as follows, which is the

same SPNE we use throughout the text:

$$a_1^* = 25, \quad a_2^*(a_1) = \begin{cases} 25 & \text{if } a_1 = OUT \\ 37 & \text{if } a_1 = 1 \\ \{33, 37\} & \text{if } a_1 = 9 \\ 29 & \text{if } a_1 = 17 \\ OUT & \text{if } a_1 = 25 \end{cases}$$

while player 3 chooses according to the following rule:

1. If the set $A = \{a_3 | v_3 > \max(v_1, v_2)\}$ is nonempty, i.e. if player 3 can attain $v_3 > \max(v_1, v_2)$ by choosing some $a_3 \in [0,1]$ he chooses one of these payoff-maximizing choices.
2. If set A is empty and the set $B = \{a_3 | v_3 = \max(v_1, v_2)\}$ is nonempty, i.e. player 3 can attain $v_3 = \max(v_1, v_2)$ by choosing some $a_3 \in [0,1]$, he chooses one of them.
3. If both sets A and B are empty, $a_3 = OUT$.

The actions that are part of the SPNE are printed in bold in Table 5. The unique subgame-perfect Nash equilibrium outcome is therefore given by $\{a_1 = 0.5, a_2 = OUT, a_3 = 0.5\}$, as all actions along the equilibrium path are unique. This result is then the same as in the sequential Hotelling game. Note that the logic behind the emergence of the unique SPNE is also very similar: As player 2 can guarantee himself a win in all subgames following $a_1 < 25$ by deterring player 3 from entering, these actions by player 1 can never be part of an SPNE. In any SPNE, the only action where player 1 is winning is $a_1 = 25$, so that is player 1's unique action on the equilibrium path.

A.3 Instructions (Baseline Treatment 24R)

Welcome to this experiment. You will be asked to make a series of choices that will affect your payoff after the experiment is over. Please pay close attention to the instructions, and do not hesitate to raise your hand in case you have any questions.

Throughout the experiment, the different payment options will be listed in Euro. In the end, you will receive the exact amount you earn in Euros.

The experiment will last for 24 rounds and will be followed by a short questionnaire. In each round, you will be in a group with two other participants, and each of you will make choices sequentially to control the biggest part of a line in order to get a payment.

Over 24 rounds, you and two other participants will form a group. All three of you will be asked to choose locations on a line. In each round, the other two members of your group will be chosen randomly, meaning that you might get completely new group members or one or two from previous rounds.

A representation of this line is given on top of the screen. The numbers on the line correspond to the locations you can choose in each round. The lowest location you can choose is 1, and the highest is 49.

A cash reward will go to the participant in a group who gets the most points in a given round. To get points, you have to control locations on the line. Each location on the line is worth a point, and the locations on the edges (1 and 49) are worth half a point, bringing the total number of points each round to 48.

The rules for how to control locations are as follows:

If you have chosen the leftmost location on the line, you control all locations to the left of you, your own chosen location and all locations that are halfway between your location and the next occupied location to your right.

If you have chosen the rightmost location, you control all locations to the right of you, your own chosen location, and all locations that are halfway between your location and the next occupied location to your left.

If your location is between two other chosen locations (meaning that your chosen location is neither the leftmost nor the rightmost), you control your own chosen locations, as well as all locations that are halfway between your location and the next occupied location to your right and all locations that are halfway between your location

and the next occupied location to your left.

In essence, this means that you control a location if it is closer to your location than to any other location chosen by your other group members. If a location is equally close to two chosen locations (for example location 8 if both location 7 and 9 have been chosen), the points for controlling this location are split equally.

You also have the option to not choose a location at all. Therefore, it is possible that three, two, one or no participant has chosen a location on the line in any given round.

Also, if two or more group members have chosen the same location, all points earned are split equally between them.

At the end of the instructions, examples will be provided to further illustrate the rules.

You and the other two members of your group make your location choice sequentially, so that you make your decision in one of three positions:

If you are in position 1, you will make your choice first. If you are in position 2, you will make your choice after observing the choice of the participant in position 1 in your group. If you are in position 3, you will make your choice after observing the choice of the participants in positions 1 and 2 in your group.

The locations you can choose from are different depending on your position: If you are in position 1, you can choose from the locations 1,9,17,25,33,41,49

If you are in position 2, you can choose from the locations 1,5,9,13,,37,41,45,49
If you are in position 3, you can choose from the locations 1,3,5,7,,43,45,47,49

This means that the later you have to make your decision, the more locations you are able to choose from.

The payoffs are as follows: Each round, you get a fixed payment of 25 Cents. If you choose a location in that round, you have to pay costs of 20 Cents. If you do not choose a location that round, you incur no costs that round.

Then, at the end of any given round, you get a payment of 2 Euros if you have the highest number of points in your group. If two or more group members have the same highest number of points, one of them gets the payment of 2 Euros randomly with equal

chance, and the others receive nothing.

Your payoff at the end of the experiment will be the sum of all payments from all rounds.

After each round, you will see a feedback screen indicating each of your group members' chosen locations, their corresponding points from controlled locations of the line, and your payment from that round.

At the beginning of each round, you will be randomly assigned two new group members as well as randomly assigned a new position. However, over the course of the experiment, you will be in each position the same number of times, meaning that you will be eight times in each of the three positions.

A Questionnaire

Describe your behavior when you were in position 1. How was your thought process behind your decisions when you were in position 1?

Describe your behavior when you were in position 2. How was your thought process behind your decisions when you were in position 2?

Describe your behavior when you were in position 3. How was your thought process behind your decisions when you were in position 3?

Imagine that you are in position 2 during the experiment, and you observe that the group member in position 1 has chosen location 17. Which location would you choose (if any), and why?

Imagine that you are in position 2 during the experiment, and you observe that the group member in position 1 has chosen location 25. Which location would you choose (if any), and why?

A.4 Regression Tests for differences between treatments

Table 6 reports on the differences between treatment 24R and 24R+A, and we see that there are at most weakly significant differences (indicated by the stars). For a detailed explanation of the regression tests used here, see footnote 26.

Table 6: *Regressions for Treatment Effects - Treatment 24R vs. Treatment 24R+A*

mean of	Treatment 24R	Constant	R^2
$a_1 = 17$.094	.614	.039
$a_1 = 25$	-.049	.261	.017
$a_2 = 33$ given $a_1 = 17$.038	.382	.006
$a_2 = 29$ given $a_1 = 17$.028	.162	.008
$a_2 = 25$ given $a_1 = 25$	-.052	.167	.039
$a_2 = OUT$ given $a_1 = 25$	-.036*	.043	.085
payoff-maximizing pl. 3	.122*	.548	.066
plays according to SPNE	-.029*	.022	.011

Notes: Each line represents a separate regression on the individual (subject) level with standard errors clustered by session; dependent variable is the mean of the binary variable on the left per subject; see footnote 26 for detailed specification; $N=72$ across all regressions (2 treatments * 12 subjects per session * 3 sessions per treatment); stars are given as follows: *: $p < 0.10$; **: $p < 0.05$; ***: $p < 0.01$.

Table 7 reports on treatment differences for rounds 1 – 24 across all four treatments. We see that there are at most weakly significant differences (indicated by the stars). The regression we use for testing has the form $Y_{i,j,k} = \beta_0 + \beta_1 * \text{Treatment48R} + \beta_2 * \text{Treatment72R} + E_{i,j,k}$, where Y is the dependent variable of interest on the left hand side, i is the subject index, j and k take values 0 or 1 (but cannot both have the value 1) and correspond to treatments 48R and 72R. Treatment 48R and Treatment 72R are dummy variables. The dependent variable is a mean that is calculated for each subject i , and standard errors are clustered by session. The null hypotheses to be tested are $\beta_1 = 0$ and $\beta_2 = 0$.

For example $Y_{3,1,0}$ would be the mean of a left hand side variable for subject 3, who was in treatment 48R, over all 24 rounds. For further examples, see footnote 26.

Table 7: *Treatment Effects for Round 1-24 - Treatment 24R and 24R+A vs. Treatment 48R and 72R: No Significant Differences*

mean of	Treatment 48R	Treatment 72R	Constant	R^2
$a_1 = 17$	-.155	-.127	.658	.097
$a_1 = 25$.130	.127	.238	.110
$a_2 = 33$ given $a_1 = 17$	-.205*	-.087	.399	.186
$a_2 = 29$ given $a_1 = 17$.047	-.040	.175	.044
$a_2 = 25$ given $a_1 = 25$.000	.014	.142	.002
$a_2 = OUT$ given $a_1 = 25$.120	.066*	.023	.214
payoff-maximizing pl. 3	.075	.099*	.609	.037
plays according to SPNE	.116	.096*	.349	.121

Notes: Each line represents a separate regression on the individual (subject) level with standard errors clustered by session; dependent variable is the mean of the binary variable on the left per subject; see footnote 26 for detailed specification (here we have two treatment dummies except for one, and both are tested); $N=132$ across all regressions (4 treatments * 12 subjects per session * 3(2) sessions per treatment); stars are given as follows: *: $p < 0.10$; **: $p < 0.05$; ***: $p < 0.01$.

Table 8 reports on treatment effects between treatments 48R and 72R, and we see that there are no significant differences. For a detailed explanation of the regression tests used here, see footnote 26.

Table 8: *Treatment Effects for Rounds 25-48 - Treatment 48R vs. Treatment 72R:*
No Significant Differences

mean of	Treatment 72R	Constant	R^2
$a_1 = 17$.034	.517	.005
$a_1 = 25$.000	.417	.000
$a_2 = 33$ given $a_1 = 17$.220	.181	.177
$a_2 = 29$ given $a_1 = 17$	-.193	.312	.236
$a_2 = 25$ given $a_1 = 25$	-.005	.073	.002
$a_2 = OUT$ given $a_1 = 25$.005	.281	.000
payoff-maximizing pl. 3	.017	.837	.002
plays according to SPNE	-.026	.576	.003

*Notes: Each line represents a separate regression on the individual (subject) level with standard errors clustered by session; dependent variable is the mean of the binary variable on the left per subject; see footnote 26 for detailed specification; N=60 across all regressions (12 subjects per session * 5 sessions); stars are given as follows: *: $p < 0.10$; **: $p < 0.05$; ***: $p < 0.01$.*

Table 9 tests whether there is a significant rise in plays of the unique SPNE in rounds 1 – 24 compared to rounds 25 – 48, and in rounds 25 – 48 compared to rounds 49 – 72. The first difference is significant, the second one is not. For a detailed explanation of the regression tests used here, see footnote 26.

Table 9: *Regression for Learning to play (25, OUT, 25)*

	Round Effect	Constant	R^2	N
Rounds 1-24 vs. 25-48	.143**	-.081	.111	228
Rounds 25-48 vs. 49-72	.143*	.063	.036	120

*Notes: Each line represents a separate regression on the individual (subject) level with standard errors clustered by session; dependent variable is fraction of times a subject was in a play of (25, OUT, 25) in any position; round effect is a dummy variable that takes the value 0 for the earlier rounds and 1 for the later rounds in each line; see footnote 26 for detailed specification; the same coefficient of .143 in both lines is not an artifact and arose by chance; stars are given as follows: *: $p < 0.10$; **: $p < 0.05$; ***: $p < 0.01$.*